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FUZZY RELIABILITY ANALYSIS AND OPTIMIZATION OF ALGORITHMIC PROCESSES

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1.Introduction

Functioning processes of many systems with discrete behaviour can be viewed from some unified positions if these are grouped into a class of the socalled ***algorithmic process (AP)***. Typical representatives of AP are information conversion processes in computer systems, processes of scientific research and design works performance, production processes, educational processes and so on. Each of these processes represents some time space detailed sequence of operations (actions, jobs) execution of which leads to goal achievement, that is to reception of some produce: information, documentation, knowledge etceteras.

Below some reliability figures for quantitative assessment are given, the necessity of which arises in design of a concrete AP:

P_{AP}- probability of correct AP performance which can be interpreted as validity of information, produce zero defects, system functioning reliability etceteras.

T_{AP} - expenses (of time or other resources) for AP performance which can be used to assess system productivity or goal achievement timeliness.

P_{AP} and T_{AP} figures assessment models are most highly elaborated in the frames of the theory of man-machine systems functioning reliability and quality [1-3]. In these works modeling is peformated on the basis of semi-Markov processes theory, the state of the processes corresponding to operators and logical conditions of the algorithm being assessed. Successful application of the AP reliability theory (by the way, as well as of the classical reliability theory) envisages the possibility of databases construction to store data about reliability characteristics of elementary operations making the process. And the problem is where from to take the source data about new operations for which there are no enough statistics and applications expertise in real conditions.

In this article we proposed the models and algorithms for AP reliability analysis and optimization using the following expert information [4,5]:

- possible ranges of operators and logical conditions reliability characteristics variations;
- fuzzy rules bases about factors influencing to the reliability characteristics.

Proposed approach is based on the fuzzy extension of probabilistic models and fuzzy logic evidence.

2. Probabilistic Models of Algorithms Reliability

For formal description of AP we get use of the Glushkov algorithmic algebra language [7]. It is customary to designate algorithm operators in this language by Latin capital letters (A, B, C...), and logical conditions - by small letters of the Greek language (α,β,γ,...) with or without indexes. According to the theorem of regularization [7], any algorithm can be represented by superposition of the following operator structures:

$B = A_1 A_2$ - linear structure consists of the process of consecutive operators A_1 and A_2 execution in the order of their registration;

$C = (A_1 \underset{\alpha}{\vee} A_2)$ - α-disjunction representing operator A execution when condition α is true (α=1), and execution of operator A_2 when condition α is false (α=0);

$D = \{A\}_{\alpha}$ - α-iteration representing cyclic execution of operator A till condition α has become true.

Let us assume that in execution of any operator A and logical condition ω the following events are possible:

$A_1 \ (A^0)$ - correct (wrong) execution of operator A;

$\omega^1 (\omega^0)$ - condition ω is true (false) a priori;

$\omega^{11} (\omega^{10})$ - a priori true condition ω recognised to be actually true (false) by checking;

$\omega^{00} (\omega^{01})$ - a priori false condition ω is a recognised to be actually false (true) by checking.

The introduced events are considered to be pairwise incompatible.

Let us designate the probability of the introduced events in this way:

$$P_A^1 \left(P_A^0\right) = \text{Prob } A^1 \left(\text{Prob } A^0 \right) \ ; \quad P_\omega \left(\overline{P}_\omega\right) = \text{Prob } \omega^1 \left(\text{Prob } \omega^0 \right) \ ;$$

$$k_\omega^{11} \left(k_\omega^{10}\right) = \text{Prob } \omega^{11} \left(\text{Prob } \omega^{10} \right) \ ; \quad k_\omega^{00} \left(k_\omega^{01}\right) = \text{Prob } \omega^{00} \left(\text{Prob } \omega^{01} \right) \ .$$

Time (or other resources) expenses for operator A and logical condition ω execution are respectively designated by T_A and T_ω .

Passing from these logical functions over to their probabilistic-expenses analogies we get the rules for algorithm execution reliability assessment:

$$B = A_1 A_2 \quad \Rightarrow \quad P_B^1 = P_{A_1}^1 \cdot P_{A_2}^1 \ , \quad T_B = T_{A_1} + T_{A_2} \ ; \tag{1}$$

$$C = \underset{\omega}{(\ A_1 \vee A_2\ )} \Rightarrow \begin{cases} P_C^1 = P_\omega\, k_\omega^{11} P_{A_1}^1 + \overline{P_\omega}\, k_\omega^{00} P_{A_2}^1 \\ T_c = T_\omega + \left(P_\omega\, k_\omega^{11} + \overline{P_\omega}\, k_\omega^{01}\right) T_{A_1} + \left(P_\omega\, k_\omega^{10} + \overline{P_\omega}\, k_\omega^{00}\right) T_{A2} \end{cases} ; \tag{2}$$

$$D = \left\{A\right\}_\omega \quad \Rightarrow \quad P_D^1 = \frac{P_A^1\, k_\omega^{11}}{1 - \left(P_A^1\, k_\omega^{10} + P_A^0\, k_\omega^{00}\right)}, \quad T_D = \frac{T_A + T_\omega}{1 - \left(P_A^1\, k_\omega^{10} + P_A^0\, k_\omega^{00}\right)} \tag{3}$$

Application of these rules allows to replace the algorithm being assessed by the only operator with equivalent characteristics of expenses and probability of correct execution.

3. Representation of indefinite source data in the form of fuzzy sets

Let q - be an indefinite parameter which corresponds to the probability of error free execution or to expenses for operator A or logical condition ω execution.

Indefinite parameter q is considered to be a linguistic variable [6], the levels of which are formalized using fuzzy sets with convex membership function given at the universal set of $U = \left[\underline{q}, \overline{q}\right]$, where $\underline{q}\left(\overline{q}\right)$, - the least (the most) possible value of parameter q. In this case the indefinite parameter q is identified with the fuzzy number $\widetilde{q}$.

Definition 1. Let us name triple

$$\widetilde{q} = \left\langle \underline{q}, \overline{q}, l \right\rangle \tag{4}$$

as l-form of an indefinite parameter q, where l - linguistic assessment of parameter q in the range of $U = \left[\underline{q}, \overline{q}\right]$ selected from the term-set $L = \left\{l_1, l_2, \ldots, l_m\right\}$, of the kind that:

$$l_j = \int_U \mu_{l_j}(q)/q,$$

where $\mu_{l_j}(q)$ - membership function of $q \in [\underline{q}, \overline{q}]$ as belonging to term $l_j \in L, j = \overline{l,m}$.

Definition 2. Let us name pairs combination

$$\tilde{q} = \bigcup_{\alpha \in [0,1]} \left(\underline{q}_\alpha, \overline{q}_\alpha\right) \tag{5}$$

as α -form of an indefinite parameter q, where $\underline{q}_\alpha \left(\overline{q}_\alpha\right)$ - the least (the most) possible value of q at α -level of membership function, that is

$$\mu\left(q_\alpha\right) = \mu\left(\overline{q}_\alpha\right) = \alpha, \quad \mu\left(\underline{q}\right) = \mu\left(\overline{q}\right) = 0.$$

Proposition 1. If membership function $\mu_{l_j}(q)$ of terms $l_1, l_2, \ldots, l_m$ are given, then *l*-form (4) can be transformed to α-form (5).

Definition 3. Let us name triple

$$\tilde{q} = \left\langle \underline{q}, \overline{q}, l(x) \right\rangle, \tag{6}$$

as the $l(x)$ -form of an indefinite parameter q in which $l(x)$ is the expert knowledge base in the form of a fuzzy logic expressions system

$$\text{IF } \left(x_1 = a_1^{j1}\right) \text{ AND } \left(x_2 = a_2^{j1}\right) \ldots \text{ AND } \left(x_n = a_n^{j1}\right) \text{ OR} \ldots$$

$$\left(x_1 = a_1^{jk_j}\right) \text{ AND } \left(x_2 = a_2^{jk_j}\right) \ldots \text{ AND } \left(x_n = a_n^{jk_j}\right), \text{ THEN } l = l_j, \tag{7}$$

where $a_i^{jp} = \int_{U_i} \mu^{jp}(x_i)/x_i, \quad i = \overline{1,n}, \; j = \overline{1,m}, \; p = \overline{1,k_j},$

which interconnect level *l* of parameter $q \in [\underline{q}, \overline{q}]$ with vector $(x_1, x_2, \ldots, x_n)$ of influencing factors, where k_j is the number of disjunction (OR) in the j-th logical expression;

$\mu^{jp}(x_i)$ - membership function of variable $x_i \in U_i$ to fuzzy term a_i^{jp}, which assesses factor x_i in the disjunction with number jp, $i = \overline{1,n}, \quad j = \overline{1,m}, \quad p = \overline{1,k_j}$.

The above given $l(x)$ -form (6) is transformed into *l*-form (4) on the basis of such proposition.

Proposition 2. To the fixed vector of $\left(x_1^*, x_2^*, \ldots, x_n^*\right)$ factors influencing parameter $q \in [\underline{q}, \overline{q}]$ corresponds such level of $l^* \in L$ of this parameter for which

$$\mu_{l^*}\left(x_1^*, x_2^*, \ldots, x_n^*\right) = \max_{j=\overline{1,m}} \left[\mu_{l_j}\left(x_1^*, x_2^*, \ldots, x_n^*\right)\right], \tag{8}$$

$$\mu_{l_j}\left(x_1^*,x_2^*,\ldots,x_n^*\right)=\bigvee_{p=1}^{k_j}\bigwedge_{i=1}^{n}\left\{\sup_{x_i\in U_i}\left[\mu^{jp}\left(x_i\right)\wedge\mu^{jp}\left(x_i^*\right)\right]\right\}. \quad (9)$$

4. Fuzzy Probabilistic Models of Algorithms Reliability

To obtain fuzzy probabilistic models from (1)-(3) we use the following extension principle.

Definition 4. If the function of n independent variables $y=f\left(q_1,q_2,\ldots,q_n\right)$ is given and arguments q_i are fuzzy numbers $\widetilde{q}_i$, given by α-form (6), **Error! Not a valid embedded object.**, them the value of function $\widetilde{y}=f\left(\widetilde{q}_1,\widetilde{q}_2,\ldots,\widetilde{q}_n\right)$ will be some fuzzy number $\widetilde{y}$ represented in a form: $\widetilde{y}=\bigcup_{\alpha\in[0,1]}\left(\underline{y}_\alpha,\overline{y}_\alpha\right)$, where

$$\underline{y}_\alpha=\inf\left(f\left(q_{1_\alpha},q_{2_\alpha},\ldots,q_{n_\alpha}\right)\right);\qquad \overline{y}_\alpha=\sup\left(f\left(q_{1_\alpha},q_{2_\alpha},\ldots,q_{n_\alpha}\right)\right);$$

$$q_{i_\alpha}=\left[\underline{q}_{i_\alpha},\overline{q}_{i_\alpha}\right],\quad i=\overline{1,n}.$$

5. Optimization of Algorithms Reliability under Fuzzinness

The problem of optimization we can formulate by the next way. It is known:

•initial variant of AP: Y=Y(A_1, A_2, ..., A_n, ω_1, ω_2, ..., ω_m);

•variants of realization of operators $A_i=\left\{A_{i_1},A_{i_2},\ldots A_{i_{ni}}\right\}$ and logical conditions $\omega_j=\left\{\omega_{j_1},\omega_{j_2},\ldots\omega_{j_{nj}}\right\}$, $i=\overline{1,n}$, $j=\overline{1,m}$;

•fuzzy probability-time characteristics of each variant of operators and conditions realizations.

It is necessary to find such variant of AP structure (vector X) which provide the best level of AP time (T) and probability of correct execution P:

$\widetilde{T}(X)\rightarrow\min\quad and\quad \widetilde{P}(X)\geq\widetilde{P}*$ - direct task of optimization;

$\widetilde{P}(X)\rightarrow\max\quad and\quad \widetilde{T}(X)\leq\widetilde{T}*$ - reverse task of optimization,

where **Error! Not a valid embedded object.** and **Error! Not a valid embedded object.** - demands to the probability and time characteristics of reliability.

To solve t

hese tasks we use the next rules of nonperspective variants cutting.

Definition 5. (The rule of fuzzy numbers compare). Let $\widetilde{a}=\bigcup_{\alpha\in[0,1]}\left(\underline{a}_\alpha,\overline{a}_\alpha\right)$ and

$\widetilde{b} = \bigcup_{\alpha \in [0,1]} \left(\underline{b}_\alpha , \overline{b}_\alpha \right)$ - are fuzzy numbers in α-forms. We will consider that

$$\begin{cases} \widetilde{a} = \widetilde{b}, \quad if \quad \forall \alpha: \ \underline{a}_\alpha = \underline{b}_\alpha \quad and \quad \overline{a}_\alpha = \overline{b}_\alpha \\ \widetilde{a} < \widetilde{b}, \quad if \quad \forall \alpha: \ \underline{a}_\alpha \le \underline{b}_\alpha \quad and \quad \overline{a}_\alpha \le \overline{b}_\alpha \quad and \quad \exists \alpha: \ \underline{a}_\alpha < \underline{b}_\alpha \quad or \quad \overline{a}_\alpha < \overline{b}_\alpha \end{cases}.$$

Proposition 3. If A_1 and A_2 are such two variants of operator A realization (A={A_1, A_2}) that $\widetilde{p}^1_{A1} > \widetilde{p}^1_{A2}$ and $\widetilde{t}_{A1} < \widetilde{t}_{A2}$, then A_2 - variant cannot to be included in the optimal AP.

Proposition 4. If ω_1 and ω_2 are such two variants of logic condition ω realization (ω={ω_1, ω_2}) that $\widetilde{k}^{11}_{\omega 1} > \widetilde{k}^{11}_{\omega 2}$, $\widetilde{k}^{00}_{\omega 1} > \widetilde{k}^{00}_{\omega 2}$ and $\widetilde{t}_{\omega 1} < \widetilde{t}_{\omega 2}$, then ω_2 - variant cannot to be included in the optimal AP.

Optimization algorithm use the scheme of brancehs and boundaries of the each step of nonperspective variants cutting.

6.Conclusions

The main difficulty of reliability theory probability models application is caused by source data lack and it doesn't allow to take into account the real conditions of system functioning. The method suggested in this article using the example of algorithm reliability assessment is one of the formal ways to solve "the problem of source data lack" on the basis of linguistic expert information and on the basis of the principle of analytical models fuzzy extension. In contradistinction to Markov process theory which is used in reliability theory the suggested method is free of complicated mathematical procedures connected with convolution of time distribution functions, the time the system being in a given state. Possible fields of application of the suggested here system of definitions and propositions are not only in reliability theory but in solutions of modelling tasks in which source data depends on many factors and the only of information about them are expert assessments.

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