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FUZZY MODEL TUNING BASED ON A TRAINING SET WITH FUZZY MODEL OUTPUT VALUES

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The paper considers the training of a fuzzy model with the help of a training set with fuzzy model output values. Two ways are proposed for constructing fuzzy rule-based multifactor models that produce fuzzy numbers at their outputs. The problem of tuning such fuzzy models on the basis of a fuzzy training set is formulated, methods of its solution are considered, and relevant examples are presented. Computational experiments showed that training based on fuzzy data improves the modeling accuracy for both crisp and fuzzy test sets.

Keywords: *tuning, fuzzy model, fuzzy inference, fuzzy training set.*

INTRODUCTION

In this article, models are considered in which the dependence "inputs-output" is described by a knowledgebase consisting of fuzzy IF—THEN rules. The tuning of a fuzzy model is an iterative procedure of changing its parameters with a view to minimizing the deviation of the results of logical inference from experimental data. The tuning of fuzzy models with knowledgebases of different formats was investigated in many works among which we note [1–9]. A distinctive feature of these investigations lies in the use of a training set with crisp values of input and output variables. Many practical identification problems in medicine, biology, economy, political science, sociology, sports, and other fields include training sets with fuzzy data. In [10], a method is proposed for training a fuzzy model with the help of a training set with fuzzy values of inputs. This article extends this method to the case of a training set with fuzzy output values. This article is organized as follows: fuzzy models transforming input numerical data into fuzzy output values are first proposed, then the tuning of a fuzzy model with the help of a fuzzy set is stated as an optimization problem, and, at the end of this article, the results of computer experiments on the tuning of the proposed models with the help of a fuzzy training set are presented.

FUZZY MODELS WITH ONE FUZZY OUTPUT

In the case being considered, a fuzzy model must realize some fuzzy function, i.e., must map crisp values of inputs $X=(x_1, x_2, \dots, x_n)$ into a fuzzy number $\tilde{y}$ at its output, $(x_1, x_2, \dots, x_n) \to \tilde{y}$. Typical models of fuzzy inference produce crisp output values. We propose two methods of synthesis of fuzzy numbers on the basis of models with fuzzy knowledgebases.

The first method consists of the elimination of the defuzzification operation from a typical fuzzy model. Then such a model based on the Mamdani-type knowledgebase produces a fuzzy set of the type

$$\tilde{y} = \int\limits_{y \in [\underline{y}, \overline{y}]} \mu_{\tilde{y}}(y) / y, \tag{1}$$

where $\mu_{\tilde{y}}(y)$ is the membership function of the fuzzy set $\tilde{y}$ whose carrier is $[\underline{y}, \overline{y}]$.

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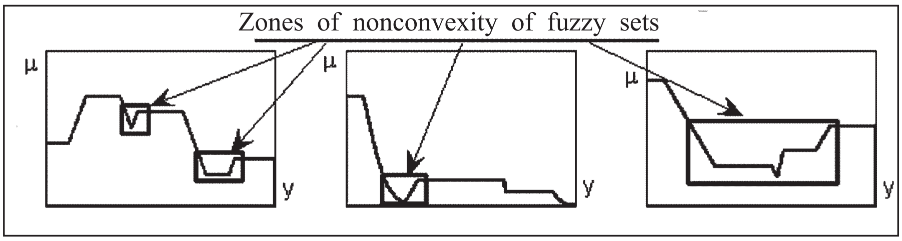

Fig. 1. Plots of the membership function of a fuzzy set that are obtained by the Mamdani algorithm.

It should be noted that fuzzy set (1) is not always equivalent to a fuzzy number since it can be subnormal or nonconvex. Any subnormal fuzzy set can be easily normalized by the division of all the grade of membership by height. The problem of correction of nonconvexity is more complicated, and we will consider it in more detail.

Typical examples of nonconvex fuzzy sets obtained by the Mamdani algorithm (Fig. 1) show that the zones of nonconvexity can include more than half of their carriers. We approximate a nonconvex fuzzy set $\widetilde{A}$ by a convex fuzzy set $\widetilde{B}$ whose membership function is parametric and that satisfies the condition

$$\left.\begin{array}{l} \text{RMSE}(\widetilde{A},\widetilde{B}) \to \min \\ \text{under the condition } \text{defuz}(\widetilde{A}) = \text{defuz}(\widetilde{B}) \end{array}\right\}, \tag{2}$$

where RMSE is the mean square discrepancy and defuz is the defuzzification of the fuzzy set by the center-of-gravity (COG) method.

We propose to compute the discrepancy of two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$ with membership functions $\mu_{\widetilde{A}}(y)$ and $\mu_{\widetilde{B}}(y)$ on the interval $[\underline{y}, \overline{y}]$ as follows:

$$\text{RMSE}(\widetilde{A},\widetilde{B}) = \sqrt{\frac{\int_{\underline{y}}^{\overline{y}} (\mu_{\widetilde{A}}(y) - \mu_{\widetilde{B}}(y))^2 \, dy}{\overline{y} - \underline{y}}}. \tag{3}$$

The plots of the membership functions presented in Fig. 1 have several lengthy plateaus. Taking into account these distinctive features in approximating nonconvex fuzzy sets, we introduce thresholds into typical membership functions to bound their values below.

The second method of synthesis of fuzzy numbers is based on the following assumptions: (1) the sought-for fuzzy number can be described by a parametric membership function of one type over the entire factor space; (2) the dependence of parameters of the membership function of this fuzzy number on influencing factors is specified by a fuzzy knowledgebase.

We introduce the following denotations: $\mu(y) = \text{mf}(y, Z)$ is a membership function (with parameters Z) on the basis of which the grades of membership $\mu(y)$ are calculated for elements y of the universal set, and $Z = f(X)$ is a fuzzy model connecting the parameters Z of the membership function with the influencing factors X, i.e., we have $\mu(y) = \text{mf}(y, f(X))$.

We describe the dependence $Z = f(X)$ by a fuzzy knowledgebase with several output variables. Each output variable corresponds to one parameter of the membership function of a fuzzy number $\widetilde{y}$.

FUZZY MODEL TUNING USING A FUZZY SET AS AN OPTIMIZATION PROBLEM

We define a fuzzy training set as M pairs of data

$$(X_r, \widetilde{y}_r), \quad r = \overline{1, M}, \tag{4}$$

where $X_r = (x_{r1}, x_{r2}, \ldots, x_{rn})$ is the input vector in the rth row of the training set and $\widetilde{y}_r = \int_{y \in [\underline{y}, \overline{y}]} \mu_{\widetilde{y}_r}(y) / y$ is the corresponding output in the form of a fuzzy number.

By analogy with [1–10], we reduce the problem of tuning based on fuzzy set (4) to the search for parameters P of a fuzzy model that satisfy the following condition:

$$\sqrt{\frac{1}{M}\sum_{r=\overline{1,M}} \mathrm{RMSE}(\tilde{y}_r, \tilde{F}(P, X_r))^2} \to \min, \tag{5}$$

where $\tilde{F}(P, X_r)$ is a fuzzy number obtained for the input vector X_r as a result of logical inference based on a fuzzy knowledgebase with parameters P and RMSE is discrepancy (3) between the desirable and actual behavior of the fuzzy model.

In the capacity of the parameters P being tuned, the parameters of the membership functions of fuzzy terms from the knowledgebase are usually used. For fuzzy Sugeno models, the coefficients in the conclusions of rules are also tuned. Problem (5) is a nonlinear optimization problem that can be solved by conventional methods.

TEST PROBLEM

In [11], data on 392 experiments are presented that contain the dependence of the time of acceleration y of a car up to the speed equal to 60 miles/hour on the number of cylinders x_1 and thrust-to-weight ratio (the power-to-weight ratio) x_2 of the car. Based on these data, we will form fuzzy training and test sets for the dependence $\tilde{y} = f(x_1, x_2)$. In the experimental data, influencing factors assume the following values: $x_1 \in \{3, 4, 5, 6, 8\}$ and $x_2 \in [0.0206, 0.0729]$. To form a fuzzy set, we round the values of the factor x_2 to thousandth. Then we obtain $x_2 \in \{0.021, 0.022, \ldots, 0.051, 0.054, 0.073\}$. The Cartesian product $x_1 \times x_2$ consists of 5*33=165 points and, among them, there are no less than three different values of the output variable y in the experimental data for 27 pairs of values (x_1, x_2). For these 27 pairs, using the potential of a point from mountain clustering [12], we compute the grades of membership from the distribution of values of the output variable. The potential of a point is a number that shows the density of experimental data at its vicinity. The higher this potential, the closer the point to the center of the corresponding cluster. The potential of a point y_i $(i = \overline{1, v})$ is calculated by the formula [12]

$$\mathrm{pot}_i = \sum_{j=\overline{1,v}} \exp(-4\alpha^2 (y_i - y_j)^2),$$

where $\alpha > 0$ is the diffuseness coefficient of the cluster and v is the number of points.

Preparatory to using this formula, we scale data by mapping them onto the unit interval. We propose to compute the grades of membership of the fuzzy set $\tilde{y}$ from potentials by the formula

$$\mu_{\tilde{y}}(y_i) = \frac{\mathrm{pot}_i}{\max\limits_{j=\overline{1,v}} (\mathrm{pot}_j)}.$$

Then we approximate the found grades of membership by the bilateral Gaussian curve

$$\mu(y) = \begin{cases} \mathrm{gmf}(y, b_1, c_1) & \text{if } y < b_1, \\ 1 & \text{if } y \in [b_1, b_2], \\ \mathrm{gmf}(y, b_2, c_2) & \text{if } y > b_2. \end{cases}$$

Here, gmf is the Gaussian membership function

$$\mu(y) = \exp\left(-\frac{(y-b)^2}{2c^2}\right), \tag{6}$$

where b and c are the parameters of the membership function (the coordinate of its maximum and coefficient of concentration). We form the training set from 20 pairs of data and the test set from seven ones (Fig. 2).

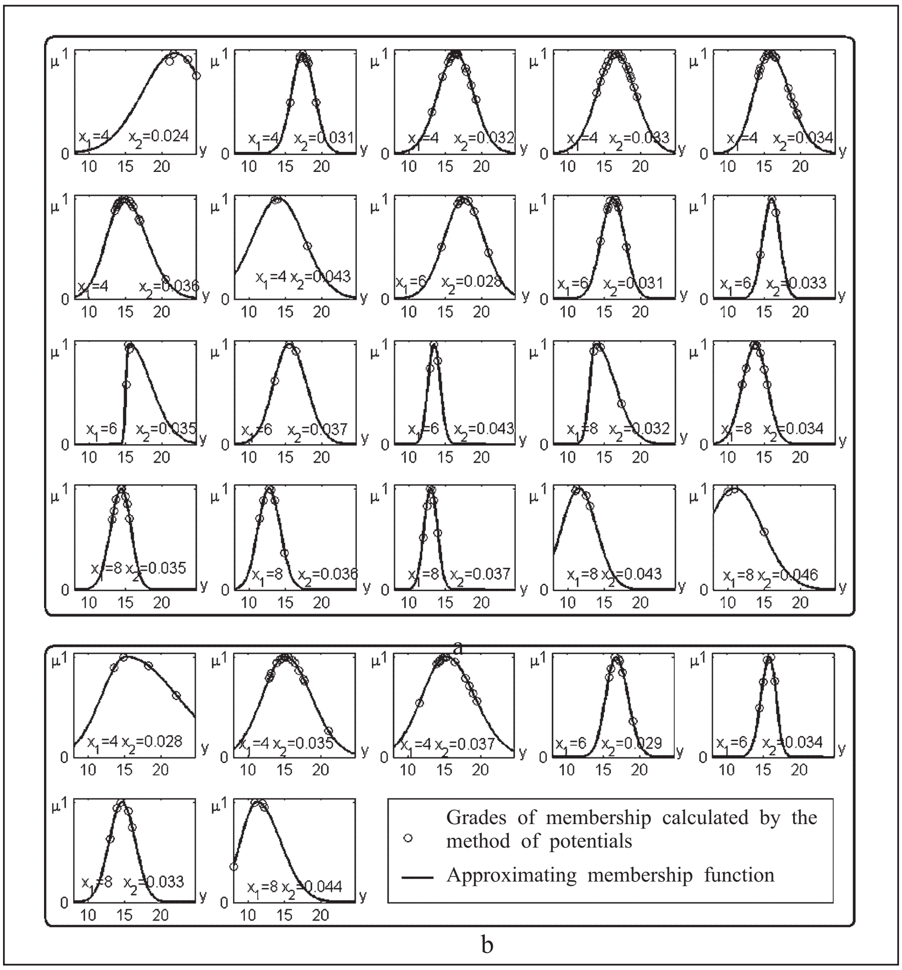

Fig. 2. Fuzzy training (a) and test (b) sets.

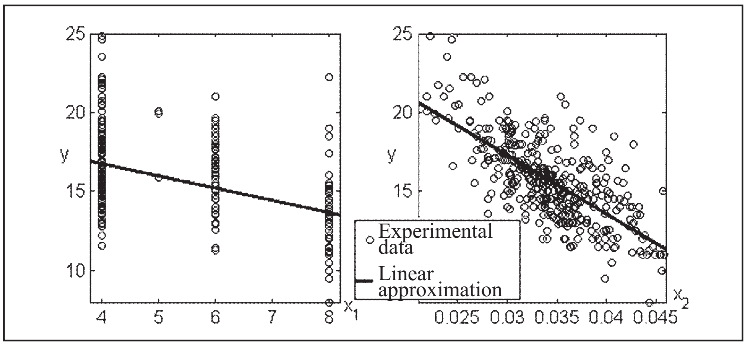

Fig. 3. Plots of distribution of experimental data from the test problem.

COMPUTER EXPERIMENTS WITH A FUZZY MAMDANI-TYPE KNOWLEDGEBASE

We form a fuzzy Mamdani-type knowledgebase from the distribution of the experimental data of the test problem (Fig. 3) according to the following rules:

if $x_1 = \tilde{4}$ and x_2 is a low thrust-to-weight ratio, then y is a long acceleration time;

if $x_1 = \tilde{6}$ and x_2 is a moderate thrust-to-weight ratio, then y is the average acceleration time;

if $x_1 = \tilde{8}$ and x_2 is a high thrust-to-weight ratio, then y is a short acceleration time.

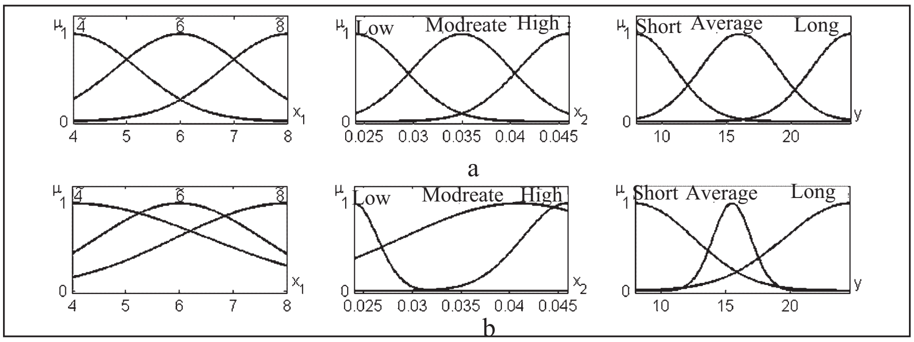

Fig. 4. Membership functions of fuzzy terms of a Mamdani-type knowledgebase before (a) and after (b) tuning.

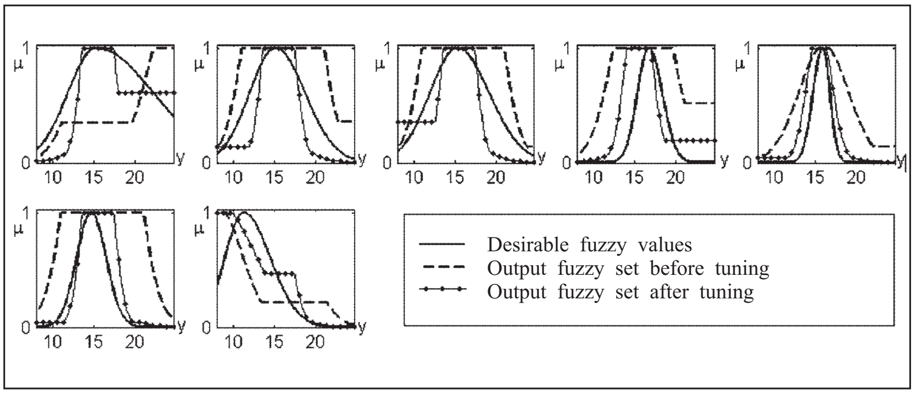

Fig. 5. Fuzzy output values of the formed Mamdani model for the test set.

TABLE 1. Fuzzy Sugeno-Type Knowledgebase

Rule	IF		THEN			
			before tuning		after tuning	
	x_1	Thrust-to-weight ratio, x_2	b	c	b	c
1	$\tilde{4}$	Low	24.6	1	24.6	2.81
2	$\tilde{6}$	Modreate	$16 + 0x_1 + 0x_2$	1	$17.73 - 0.23x_1 - 0.03x_2$	1.84
3	$\tilde{8}$	High	8	1	8	7.23

We construct membership functions of fuzzy terms from Gaussian curve (6). The plots of the membership functions before and after tuning are shown in Fig. 4. Testing (Fig. 5) testifies to the fact that, as a result of tuning, discrepancy (5) for the fuzzy test set has reduced from 0.4338 to 0.2194.

COMPUTER EXPERIMENTS WITH A FUZZY SUGENO-TYPE MODEL

For the test problem, we construct a fuzzy model with two outputs that correspond to the parameters b and c of the Gaussian membership function of the fuzzy acceleration time. We form the knowledgebase of the fuzzy Sugeno model according to three rules from Table 1. The accuracy of prediction of this model with the membership functions presented in Fig. 4a for the fuzzy test set equals $\text{RMSE} = 0.4284$. After tuning, the accuracy has increased in comparison with the previous model to $\text{RMSE} = 0.2158$. The results of testing are given in Fig. 6. The optimized membership functions are shown in Fig. 7, and conclusions of rules are presented in Table 1. The negative coefficients in the conclusion of the second rule testify to the fact that, during tuning, the tendency arises of reducing the acceleration time with increasing the number of cylinders and thrust-to-weight ratio of the car, which corresponds to the distribution of experimental data in Fig. 3.

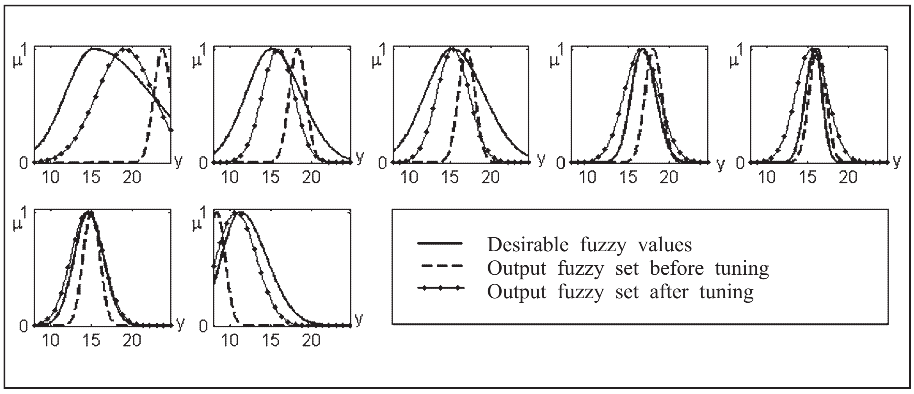

Fig. 6. Fuzzy output values of the Sugeno model for the test set.

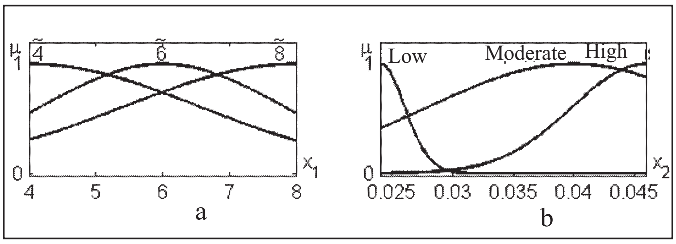

Fig. 7. Optimized membership functions of fuzzy terms of a Sugeno-type knowledgebase.

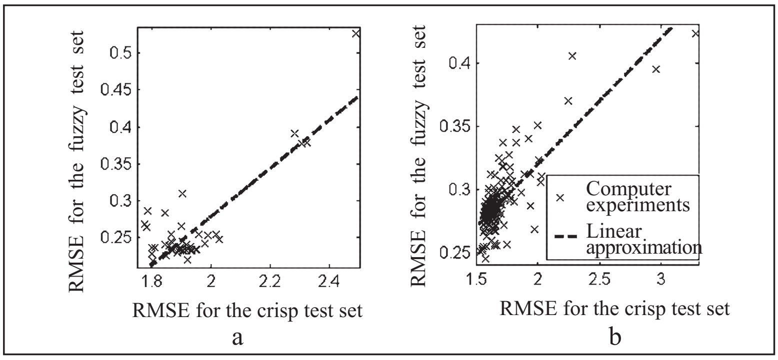

Fig. 8. Comparison of discrepancies obtained for fuzzy and crisp test sets after model tuning using a fuzzy training set for the Mamdani model (a) and Sugeno model (b).

FUZZY MODEL TESTING USING CRISP TEST SETS

To determine the possibility of identification of crisp multifactor dependences with the help of a training set with fuzzy output values, the correspondence between the discrepancies obtained for fuzzy and crisp test sets was checked. To this end, fuzzy models were tuned using a fuzzy training set consisting of different initial points. After optimization, they were tested with the help of fuzzy and crisp test sets. During testing, the crisp test set was defuzzified by the center-of-gravity method. The results of testing presented in Fig. 8 testify that the discrepancies obtained for crisp and fuzzy data are correlated and, hence, the fuzzy model tuning using a fuzzy training set can be considered as a way of identification of multifactor dependences from fuzzy initial information.

CONCLUSIONS

Two methods of construction of multifactor models are considered that are based on fuzzy knowledgebases and produce fuzzy numbers at their outputs. The first method consists of elimination of the defuzzification operation from a fuzzy Mamdani model with subsequent approximation of its fuzzy output set by a parametric membership function. In the second method, a fuzzy value at the output of a model is specified by a membership function whose parameters depend on influencing factors. The dependence of the influencing factors on the parameters of the membership function is specified by a fuzzy knowledgebase. A statement of the problem of tuning of such fuzzy models with the help of a fuzzy training set is proposed. A distinctive feature of this problem statement is that the concept of a training set with fuzzy values of the output variable is first introduced. Computer experiments showed that the tuning using fuzzy data increases the accuracy of modeling for both fuzzy and crisp data sets. Owing to the possibility of using a fuzzy training set, the approach proposed will be useful in identifying multifactor dependences in problems for which available experimental data are represented by fuzzy numbers.

REFERENCES

1. T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” IEEE Trans. on Systems, Man, and Cybernetics, **15**, No. 1, 116–132 (1985).
2. J.-S. R. Jang, “ANFIS: Adaptive-network-based fuzzy inference system,” IEEE Trans. Syst., Man, Cybern., **23**, 665–685 (1993).
3. A. Rotshtein, “Design and tuning of a fuzzy rule-based system for medical diagnosis,” in: N. H. Teodorescu, A. Kandel, and L. C. Jain (eds.), Fuzzy and Neuro-Fuzzy Systems in Medicine, CRC-Press, Boca-Raton, USA (1998), pp. 243–289.
4. A. P. Rotshtein and D. I. Katelnikov, “Identification of nonlinear dependences by fuzzy knowledgebases,” Cybernetics and Systems Analysis, No. 5, 53–61 (1998).
5. A. P. Rotshtein, Intelligent Identification Technologies: Fuzzy Logic, Genetic Algorithms, and Neural Networks [in Russian], UNIVERSUM-Vinnytsya, Vinnitsa (1999).
6. A. P. Rotshtein and S. D. Shtovba, “Influence of defuzzification methods on the rate of tuning a fuzzy model,” Cybernetics and Systems Analysis, No. 5, 169–176 (2002).
7. S. Shtovba, A. Rotshtein, and O. Pankevich, “Fuzzy rule-based system for diagnosis of stone construction cracks of buildings,” in: H-J. Zimmermann, G. Tselentis, M. van Someren, and G. Dounias (eds.), Advances in Computational Intelligence and Learning: Methods and Applications, Kluwer, Dordrecht (2002), pp. 401–412.
8. S. Shtovba, O. Pankevich, and G. Dounias, “Tuning fuzzy classification models with various learning criteria: The case of credit data classification,” in: Proc. Inter. Conference on Fuzzy Sets and Soft Computing in Economics and Finance, **1**, St. Petersburg, Russia (2004), pp. 103–110.
9. A. P. Rotshtein, M. Pozner, and A. B. Rakityanskaya, “Football predictions based on a fuzzy model with genetic and neural tuning,” Cybernetics and Systems Analysis, No. 4, 171–184 (2005).
10. A. P. Rotshtein and S. D. Shtovba, “Identification of a nonlinear dependence by a fuzzy knowledgebase in the case of a fuzzy training set,” Cybernetics and Systems Analysis, No. 2, 17–24 (2006).
11. MPG Data Base of UCI Machine Learning Repository, http://www.ics.uci.edu/~mlearn/MLRepository.html.
12. R. Yager and D. Filev, Essentials of Fuzzy Modeling and Control, Wiley, New York (1994).