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Ensuring Accuracy and Transparency of Mamdani Fuzzy Model in Learning by Experimental Data

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ABSTRACT

Typical violations of transparency of the Mamdani fuzzy model, which arise as a side effect of learning by experimental data are revealed. We suggest a new learning scheme of the Mamdani fuzzy model, which differs from the known ones by the following: 1) expansion of supports of fuzzy sets of output variable; 2) excluding coordinates of maxima of membership functions of extreme terms from the list of parameters to be tuned; 3) introducing constraints for linear ordering of fuzzy sets within limits of one term-set. Computer simulations indicate that learning by the new scheme does not break transparency of a fuzzy model. Moreover, accuracy of fuzzy model is not worse than for the case of typical learning.

Key words: Mamdani fuzzy model, accuracy, transparency, learning scheme, membership function, knowledge base, optimization problem, computer simulation.

Introduction

We consider models, where dependence between inputs and outputs is described by a knowledge base from fuzzy rules of the type "if – then". In fuzzy modeling, a knowledge base in the format of Mamdani is often used. Here, antecedents and consequents of the rules are prescribed by fuzzy sets, for example, "Low", "Average", "High" etc. Fuzzy rules in such format were suggested in paper [1], on the basis of which E. Mamdani and S. Assilian developed the first fuzzy controller [2]. Unlike models of the "black box" type, Mamdani fuzzy models are transparent; their structure has a meaningful interpretation in terms, which are clear not only to developers with a high mathematical qualification, but as well to customers: doctors, economists, managers. Transparency of Mamdani fuzzy models is one of the most important advantages, thanks to which fuzzy technologies successfully compete with other methods, especially for those applied problems, where possibility of meaningful interpretation is more important, than modeling accuracy.

For enhancing accuracy a fuzzy model is learned, i.e., its parameters are iteratively changed to minimize deviation of logical inference results from experimental data. Both weights of the rules and membership functions of fuzzy terms are tuned. Learning of a Mamdani fuzzy model is a problem of nonlinear optimization, and numerous theoretical and applied works are devoted to its study. These works make the principal stress on attaining maximal accuracy of a fuzzy model learning. In doing so, tuned parameters sometimes change so much, that some complexities of meaningful interpretation of fuzzy model arise. Thus, "rush for accuracy" results in loss of an important competitive advantage, transparency of a fuzzy model. If model transparency is of secondary importance, then in identification of dependencies it is more expedient to use other (nonfuzzy) methods, which are as a rule more adequate [3].

The goal of this paper is to study a new technique of preserving transparency of Mamdani fuzzy model and enhancing its accuracy in learning by experimental data. First we describe the Mamdani fuzzy model, formalize model learning in the form of a problem of nonlinear optimization and analyze ways of enhancing learning accuracy, then we formulate requirements of transparency of Mamdani fuzzy model and analyze main methods of its preserving. For enhancing accuracy of this model we suggest tuning additional parameters, boundaries of the supports of fuzzy sets in consequents of rules and for preserving its transparency we reduce the number of controlled variables and introduce new constraints. The typical and the suggested learning schemes are compared on the example of prognosing fuel efficiency of a car.

1. Fuzzy Mamdani model

Fuzzy Mamdani knowledge base is written down as follows [1, 2, 4, 5]:

$$\text{if } \left(x_1 = \tilde{a}_{1j} \text{ and } x_2 = \tilde{a}_{2j} \text{ and} \ldots \text{and } x_n = \tilde{a}_{nj} \text{ with weight } w_j\right), \text{ then } y = \tilde{d}_j, \quad j = \overline{1,m}, \tag{1}$$

where $\tilde{a}_{ij}$ – is the fuzzy term, estimating the variable x_i in the *j*-th rule, $i = \overline{1,n}$;

$\tilde{d}_j$ – is fuzzy conclusion of the *j*-th rule;

m – is the number of rules in the knowledge base;

$w_j \in [0, 1]$ – is the weight coefficient reflecting the adequacy of the *j*-th rule.

Mamdani knowledge base can be treated as partition of the feature space by affecting rules into zones with fuzzy boundaries, inside which the response function takes fuzzy value. The number of such zones is equal to the number of rules.

Let us introduce the following notations, necessary for further explanation:

- $\mu_j(x_i)$ – is membership function of input $x_i \in \left[\underline{x}_i, \overline{x}_i\right]$ to the fuzzy term $\tilde{a}_{ij}$, i.e., $\tilde{a}_{ij} = \int\limits_{x_i \in \left[\underline{x}_i, \overline{x}_i\right]} \mu_j(x_i)/x_i$;
- $\mu_{d_j}(y)$ – is membership function of output $y \in \left[\underline{y}, \overline{y}\right]$ to the fuzzy term $\tilde{d}_j$, i.e., $\tilde{d}_j = \int\limits_{y \in \left[\underline{y}, \overline{y}\right]} \mu_{d_j}(y)/y$.

Degree of fulfillment of the *j*-th rule antecedent for the current input vector $X^* = \left(x_1^*, x_2^*, \ldots, x_n^*\right)$ is calculated as follows:

$$\mu_j(X^*) = w_j \cdot \left(\mu_j(x_1^*) \wedge \mu_j(x_2^*) \wedge \ldots \wedge \mu_j(x_n^*)\right), \quad j = \overline{1,m},$$

where $\wedge$ is *t*-norm, which in Mamdani inference is usually implemented by the operation of minimum.

In the result of logical inference by the *j*-th rule of knowledge base we obtain the following value of output variable y:

$$\tilde{d}_j^* \quad \mathrm{imp}\left(\tilde{d}_j,\ \mu_j(X^*)\right), \quad j \quad \overline{1,m}, \tag{2}$$

where imp – is the implication, which in Mamdani inference is implemented by the operation of minimum, i.e., by "shearing" the membership function $\mu_{d_j}(y)$ by the level $\mu_j(X^*)$. Mathematically this is written as follows:

$$\tilde{d}_j^* \quad \int\limits_{y\in\left[\underline{y},\,\overline{y}\right]} \min\left(\mu_j(X^*),\ \mu_{d_j}(y)\right)/\,y.$$

The resulting fuzzy set $\tilde{y}^*$ is obtained by uniting fuzzy sets (2):

$$\tilde{y}^* \quad \tilde{d}_1^* \cup \tilde{d}_2^* \cup \ldots \cup \tilde{d}_m^*,$$

which corresponds to the operation of maximum over the membership functions:

$$\mu_{y^*}(y) \quad \max\left(\mu_{d_1^*}(y), \mu_{d_2^*}(y), \ldots, \mu_{d_m^*}(y)\right).$$

A crisp value of output y^*, corresponding to the input vector X^*, is defined by defuzzification of fuzzy set $\tilde{y}^*$. Most often the defuzzification is carrying out by the centroid method:

$$y^* \quad \frac{\int_{\underline{y}}^{\overline{y}} y\mu_{y^*}(y)\,dy}{\int_{\underline{y}}^{\overline{y}} \mu_{y^*}(y)\,dy},$$

which ensures the best dynamics of learning the fuzzy model [6].

2. Problem of learning a Mamdani fuzzy model

The training set about inputs-output mapping for the studied dependence is denoted as

$$(X_r, y_r), \qquad r \quad \overline{1,M}, \tag{3}$$

where $X_r \quad (x_{r1}, x_{r2}, \ldots, x_{rn})$ – is the input vector in the *r*-th pair of data and y_r – is the corresponding output; M – is the length of the training set.

For mathematical formulation of problem of fuzzy model learning by training set (3), introduce the following notations:

P – is the vector of membership function parameters for terms of input and output variables;

W – is the vector of rule weights of the knowledge base;

$F(P,W,X_r)$ – is the result of inference by a Mamdani fuzzy knowledge base with parameters (P,W) for values of inputs X_r. Fuzzy inference is performed by formulae from the preceding Section of this paper.

According to [4-8], learning of fuzzy model consists in finding a vector (P,W) such, that

$$RMSE(P,W) \quad \sqrt{\frac{1}{M} \sum_{r \quad \overline{1,M}} (y_r - F(P,W,X_r))^2} \rightarrow \quad \min . \qquad (4)$$

In tuning membership functions the following five methods are applied. According to them the coordinates of vector P are prescribed as follows:

1) coefficients of parametric membership function of each fuzzy term [4-8]. For instance, triangle membership function is prescribed by three coefficients, which correspond to the core and the support boundaries of the fuzzy set. The length of vector P is calculated as follows: $|P| \quad \sum_{u \quad \overline{1,N}} k_u$, where k_u – is the number of coefficients of membership function of the u-th fuzzy term; N – is the number of fuzzy terms in the knowledge base (1);
2) boundaries of α-cuts of each fuzzy term [7]. In this case $|P| \quad 2 \sum_{u \quad \overline{1,N}} h_u$, where h_u –is the number α-cuts for u-th fuzzy term. For instance, for prescribing α-cuts at levels 0, 0.5 and 1 we need six coefficients – $(left_0, right_0, left_{0.5}, right_{0.5}, left_1, right_1)$ (see Figure 1);
3) linguistic hedges "very", "more or less" and so on [9, 10]. In this case $|P| \quad N$. As usual the hedge "very" concentrates a fuzzy set and the hedge "more or less" dilates it. Operations of concentration and dilation of fuzzy sets are implemented by exponentiation of the membership function to degrees 2 and ½ respectively [1];
4) coefficients of concentration and dilation of fuzzy terms [9]. These coefficients correspond to, exponents applied to membership function. Influence of this coefficient (β) on the membership function curve is illustrated by Figure 2. Values of β=2 и β= ½ are equivalent to hedges "very" and "more or less";
5) coefficients of the membership function, linguistic hedges and the coefficient of concentration and dilation for each fuzzy term [9]. This hybrid approach unites techniques 1, 2 and 4. In this case dimension of optimization problem (4) significantly increases, since the number of tuned coefficients of each fuzzy term increases by two compared to the first technique.

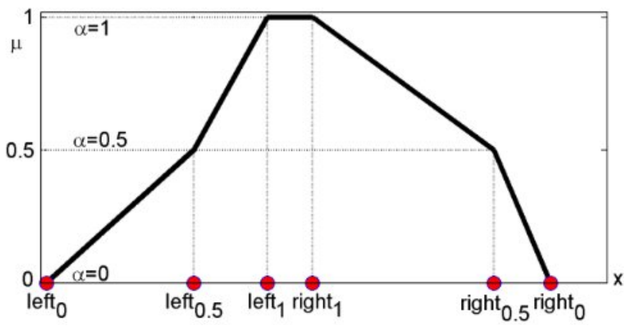

Figure 1

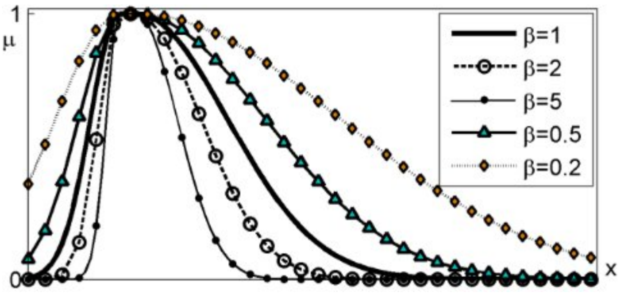

Figure 2

Comparative analysis of techniques of tuning the membership functions is given in Table 1. Most often the first technique of tuning the membership function is used, thanks to a good balance between accuracy and duration of learning fuzzy model. This is the technique used in this paper below.

Table 1

Technique	Advantages	Disadvantages
1	Differentiable goal function	Type of membership function in tuning cannot be changed
2	Differentiable goal function No analytical model of membership function is required Potentially attainable accuracy of learning is attainable	Large dimension of optimization problem (4) Many constraints for controlled variables, necessary for ensuring convexity of fuzzy sets
3	Minimal dimension of optimization problem (4)	Cores of fuzzy sets are not tuned Discrete optimization problem with a small number of alternatives, which provides no opportunity for accurate tuning of fuzzy model
4	Minimal dimension of optimization problem (4) Differentiable goal function High accuracy of learning is potentially attainable	Cores of fuzzy sets are not tuned
5	High accuracy of learning is potentially attainable	Mixed discrete-continuous optimization problem of large dimension.

3. Transparency of fuzzy model

We shall call a fuzzy Mamdani model transparent under fulfillment of such conditions:

1) knowledge base is not contradictory or excessive, i.e., contains no rules with the same antecedents;

2) knowledge base is consistent with the number of terms, i.e., each term appears at least in one fuzzy rule;

3) for any input vector, at least one nonempty fuzzy set is obtained at output;

4) taken separately, each membership function has a substantive interpretation, i.e., the corresponding fuzzy set is normal and convex [11, 12];

5) each term-set has a substantive interpretation, i.e.:

- the number of terms is not too large, so expert is able to put into correspondence a linguistic score [11 – 13] to each fuzzy set. Following works [7, 14] it is expedient to bound power of a term-set from above by a "magic" number 7 ± 2 [15];
- fuzzy sets of different terms should not be equivalent or almost equivalent [11 – 13]. Hence, plots of membership functions of neighboring terms, for example, "Low" or "Below average" should be visually distinguished;
- linear ordering of fuzzy sets should be obeyed, i.e., for term-sets {"*Low*", "Below average",…, "High"} of variable *x* we have:

$$\forall x: \; Low \leq Below\,Average \leq \ldots \leq High. \tag{5}$$

Besides, it is desirable, that the knowledge base would be compact, i.e., contain minimal (or close to it) number of rules necessary for adequate modeling of the studied dependence. For a large number of input variables the knowledge base compactness ensures hierarchical representation of rules [7, 14].

After tuning the membership functions, the following typical violations of transparency of fuzzy model arise (Figure3):

a) strong similarity of membership functions of neighboring fuzzy sets ("Low" and "Below Average" in Figure 3), which can bring contradictions to the knowledge base;

b) loss of linear ordering of a term-set due to different spreading of membership functions, e.g., in interval [65, 82] the fuzzy set "Average" is larger, than the fuzzy set "Higher average" and in the interval [0, 3] the fuzzy set "Below average" is larger than the fuzzy set "Low", though it should be vice versa;

c) impossibility of representation of the outermost term, decreasing values of variable *x* from 8 to 0 reduces degree of membership to the fuzzy term "Low", though it should be vice versa;

d) incomplete covering by fuzzy sets of interval of possible values of input variables, numbers from the range [82, 88] do not belong to fuzzy sets.

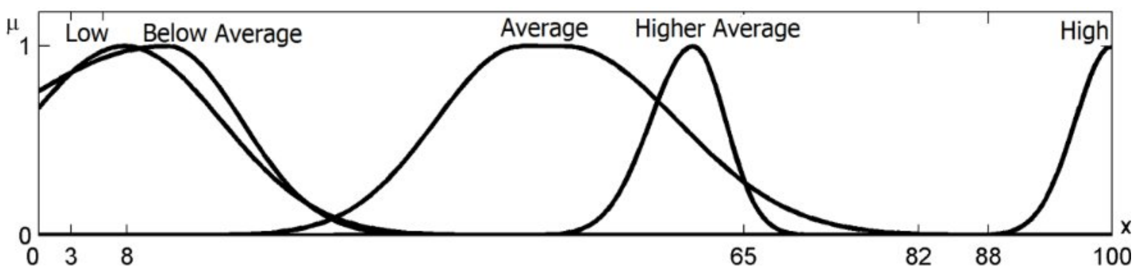

Figure 3

4. Methods of protecting transparency of fuzzy model

The first two conditions of transparency of a fuzzy model refer to the knowledge base. Let us assume that they hold during its forming and in the course of learning the rules do not change.

The third condition can be fulfilled using fuzzy terms, whose supports are not narrower, than the range of variation of the corresponding input variable. In so doing, the simplest way is to use parametric membership functions with the domain of definition $(-\infty, \infty)$, for example, Gaussian curve:

$$\mu(x) \quad \exp\left(-(x-b)^2/2c^2\right), \tag{6}$$

where b – is the coordinate of maximum; $c > 0$ – is the coefficient of concentration.

The fourth condition can be easily fulfilled with parametric membership functions, which prescribe convex and normal fuzzy sets. In so doing we should bound cores of fuzzy sets by the range of variation of the corresponding variable. For example, for the membership function (6) this constraint is written as: $b \in [\underline{x}, \overline{x}]$.

For fulfillment of condition 5 we can apply three approaches, which use: 1) formal criterion of transparency; 2) linguistic quantifiers for modification of membership functions; 3) semantic constraints.

In the first approach we synthesize some criterion of transparency $T(P)$ of fuzzy model and pass from (4) to the following multiobjective optimization problem [16]: to find the vector (P, W), such, that:

$$\begin{cases} RMSE(P, W) \to \min \\ T(P) \to \max \end{cases}. \tag{7}$$

For solving problem (7) by typical methods, an integral criterion [12] is synthesized. We also can transform one of criteria into a constraint, which transforms (7) into a problem of constrained optimization.

Calculation of transparency of a fuzzy model is usually performed on the basis of the following coefficient of similarity of fuzzy sets [12, 13, 16]:

$$S(\tilde{A}, \tilde{B}) \quad \frac{|\mathrm{supp}(\tilde{A} \cap \tilde{B})|}{|\mathrm{supp}(\tilde{A} \cup \tilde{B})|}, \tag{8}$$

where $\tilde{A}$ and $\tilde{B}$ – are fuzzy sets, for which the similarity coefficient is calculated; supp – is the support of fuzzy set; $|.|$ – is the power of the set. The value of the similarity coefficient is equal to 1, if fuzzy sets are equivalent, and is equal to 0, if fuzzy sets do not intersect. Formula (8) is applicable to calculation of similarity coefficients of fuzzy sets with compact supports. If the support of fuzzy sets is the interval $(-\infty, \infty)$, then formula (8) is inapplicable. In this case we can modify it, replacing bounds by α - cuts:

$$S(\tilde{A}, \tilde{B}) \quad \frac{|(\tilde{A} \cap \tilde{B})_\alpha|}{|(\tilde{A} \cup \tilde{B})_\alpha|},$$

where $\alpha << 1$, for instance, $\alpha \quad 0.01$.

As the index of transparency some researchers select quantity inverse to the mean or the maximal similarity coefficient over all pairs of fuzzy sets from the knowledge base. Using the coefficient (8) protects from only one violation of fuzzy model transparency, namely similarity of membership functions of fuzzy terms. Therefore its isolated use does not preserve transparency of model.

In the second approach membership functions are tuned by changing linguistic hedges of fuzzy sets [9, 10]. In so doing the original membership functions are selected so that any combination of hedges would not break transparency of term-sets. Drawback of this approach is low accuracy of tuning a fuzzy model.

In the third approach the following constraints for values of controlled variables are applied:

(i) fuzzy sets completely cover the interval of possible values of input variables [11], i.e., any number from this interval belongs with a nonzero degree to at least one fuzzy set; (9)

(ii) coordinates of maxima of membership functions of fuzzy terms are bounded by ranges of variation of corresponding variables [5, 7, 11]; (10)

(iii) only membership functions of neighboring fuzzy terms [11] have intersections; (11)

(iv) height of intersection of fuzzy sets of neighboring terms is bounded from below and from above [17]; (12)

(v) distance between coordinates of maxima of membership functions of neighboring terms is bounded from below [5, 13]; (13)

(vi) the length of a certain α-cut of each fuzzy set is bounded from below and from above [16]. (14)

The constraints (i)-(vi) can be used either jointly or individually. However, under fulfillment of even all constraints breaks of linear ordering of a term-set and loss of interpretability of membership functions of extreme terms are possible. Therefore the author of [17] suggests using new constraints which can be formulated as requirements of fuzzy Ruspini partition of interval of input values:

$$\forall x_i \in \left[\underline{x}_i, \overline{x}_i\right]: \quad \sum_{v = \overline{1,V_i}} \mu_{iv}(x_i) = 1,$$

where V_i – is the power of term-set of variable x_i, $i = \overline{1,n}$. With the use of Ruspini partition together with triangle membership functions one can tune only cores of non-extreme fuzzy sets, since remaining coefficients are related to them (Figure 4). Such reducing the tuned coefficients diminishes accuracy of learning of fuzzy models [9]. Besides, Ruspini partition is applicable only to fuzzy sets with compact supports, for instance, those prescribed by a triangle or a trapezoidal membership functions. The use of such membership functions increases volume of knowledge base, since for arbitrary input vector there should exist at least one rule with nonzero degree of fulfillment. Authors of paper [9] use the following constraint together with (i)-(iii):

(vii) the lower and the upper boundaries of the core and the support of each fuzzy term are bounded from below and from above. (15)

Under such constraints we can obtain more accurate fuzzy models compared to the condition of Ruspini partition. However, in learning interpretability of extreme terms is not preserved. One more drawback, like in the case of using fuzzy partition, is large volume of knowledge base.

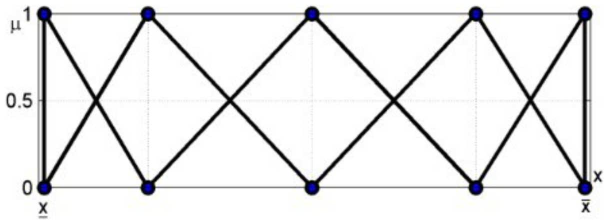

Figure 4

Thus, hard constraints in the form of Ruspini partition preserve transparency of a fuzzy model in learning, however they significantly reduce accuracy. The analyzed attempts to soften these constraints enhance accuracy of learning, but result in different breaks of transparency. Below we suggest a new approach to learning the membership functions, which enhances accuracy of Mamdani fuzzy model without sacrifice of transparency.

5. Enhancing accuracy of Mamdani model learning via the extension of support of fuzzy sets

In majority of Mamdani fuzzy models, defuzzification is performed by the centroid method. For such fuzzy models, the effect of narrowing range of output values (Figure 5, a) was revealed. The effect consists in the following. The minimal value at the output of Mamdani fuzzy model will be obtained, when the degree of fulfillment of the rule with the consequent "Low" is equal to 1, and degrees of fulfillment of remaining rules are equal to 0. In this case the result of logical inference is calculated by means of defuzzification the fuzzy set "Low". Analogously, the maximal output value will be the result of defuzzification the fuzzy set "High". The more smeared are fuzzy sets "Low" and "High", the more distant are results of defuzzification from coordinates of maxima of membership functions and, respectively, from required boundaries of output values.

We can eliminate the effect of narrowing range of output variables without sacrifice of transparency by expansion of the support of fuzzy sets (Figure 5, *b).* Thus, accuracy of learning Mamdani model, beside parameters listed in Section 2, is affected by boundaries of the support of fuzzy sets of output variable. Therefore it is expedient to try to enhance the accuracy of learning the fuzzy model by adding these two parameters to the vector of controlled variables of optimization problem (4).

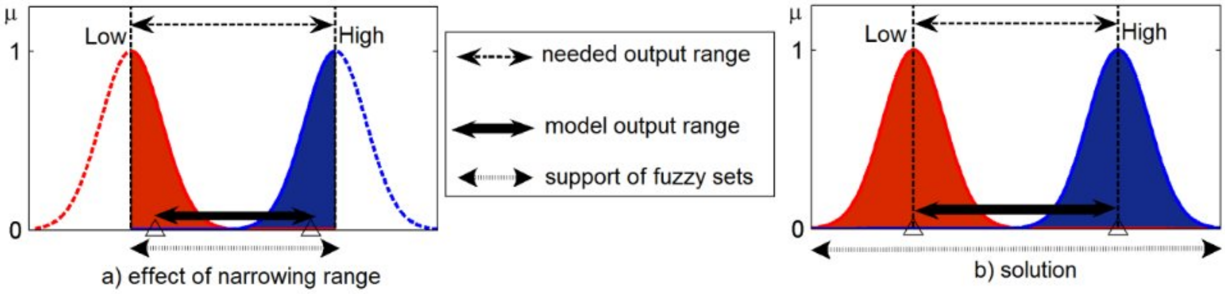

Figure 5

Example 1. Consider the dependence (Figure 6) of MPG - miles per gallon of fuel (y) on the automobile mass (x_1) and the year of automobile manufacture (x_2) [18]. The fuzzy Mamdani model of the dependence $y \quad f(x_1, x_2)$ is given in Table 2 and Table 3. Gaussian membership function (6) is used.

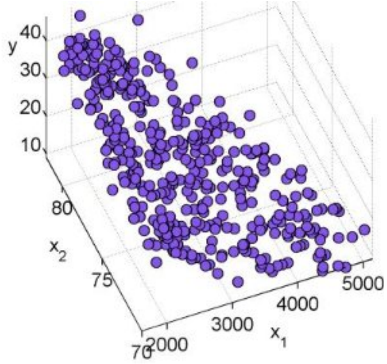

Figure 6

Table 2

No. of rule	x_1	x_2	y
1	Light	**New**	High
2	Heavy	Old	Low
3	Light	Old	Average

Table 3

Fuzzy term	b	c
Light	1613	1500
Heavy	5140	1500
Old	70	6
New	82	3
Low	9	5
Average	30	5
High	46.6	5

Testing (Figure 7, *a*, source fuzzy model) demonstrates, that fuzzy model is performing badly for automobiles with very small and very high fuel efficiency. Extension of the support of fuzzy sets of the output variable *y* significantly enhances the accuracy (Figure 7, *b*, the model with the expanded support of fuzzy sets of the output variable and Figure 8).

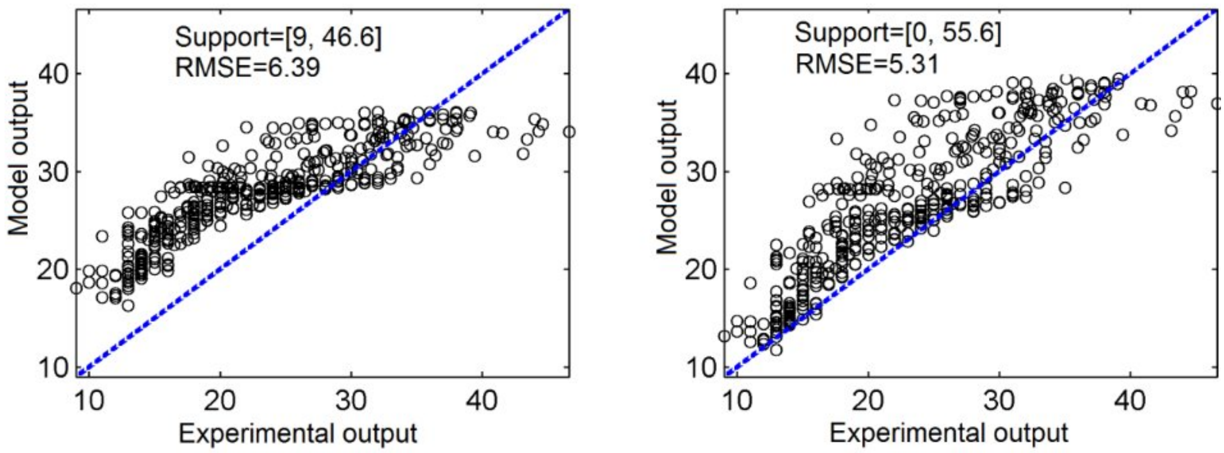

Figure 7

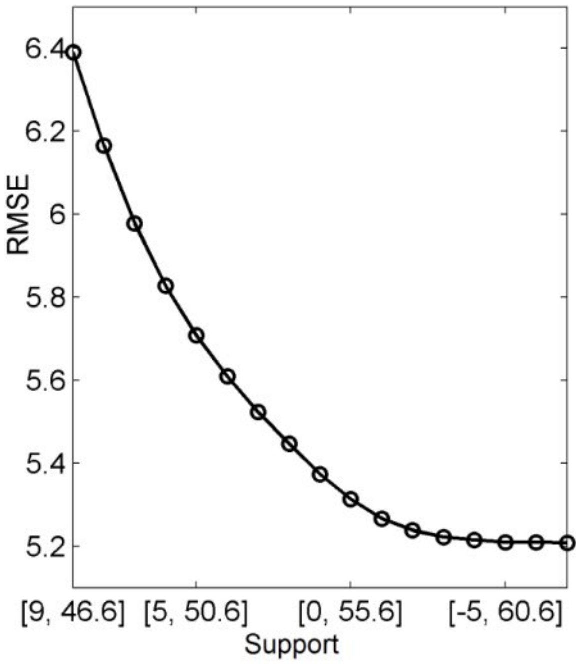

Figure 8

6. New scheme of preserving the transparency of Mamdani fuzzy model

For ensuring a compactness of knowledge base we use fuzzy sets with a noncompact support, for instance, with Gaussian membership function (6). For preventing narrowing the range of output values we include boundaries of the support of fuzzy sets of the output variable into the vector of controlled variables of problem (4). For preserving transparency of fuzzy model we introduce the constraints (5), (i) and (v) into optimization problem (4). The constraints (5) and (i) can be implemented in the simplest way, if we select ranges of variation of concentration coefficients *(c)* of Gaussian membership function (6). Moreover, we shall not tune coordinates of maxima of the extreme terms, but set them equal to boundaries of possible values of variables. This will prevent us from loss of noninterpretability of membership functions of extreme fuzzy terms, and also reduce the dimension of the optimization problem (4). We shall use weight coefficients only for those rules, whose adequacy raises doubts. Values of remaining weight coefficients are set to 1. The reduction of the number of tuned parameters, besides diminishing the time of optimization, enables us also to reduce size of the training sampling of experimental data.

Example 2. Let us compare results of learning the fuzzy model from the previous example by the new and the typical schemes. In typical learning scheme we have 17 controlled variables:

- three weight coefficients of rules of the knowledge base;
- seven concentration coefficients of membership functions of fuzzy terms;
- seven coefficients of maxima of membership functions of fuzzy terms.

Learning is performed with constraints (i), (ii) and (v).

In the new learning scheme we have 11 controlled variables:

- the weight coefficient of the third rule of knowledge base since adequacy of the first and the second rules is evident;
- the left and the right boundaries of the support of fuzzy sets of the output variable y;
- seven concentration coefficients of membership functions of fuzzy terms;
- the coordinate of maximum of the membership function of fuzzy term "Average".

Let us include into the training set automobiles with odd serial numbers, and into the test set – those with even numbers. Learning is performed by quasi-Newtonian method of BFGS (Broyden-Fletcher-Goldfarb-Shanno) on 15 iterations.

Distributions of errors on the test set (statistics of 100 experiments) (Figure 9) under optimization from different starting points demonstrates that learning by the new scheme is in average more accurate. The membership functions and the weight coefficients of rules of the best models obtained by learning by the both schemes are shown in Figure 10 and in Table 4. The errors on the test set are: RMSE=2.930 – after learning by the typical scheme and RMSE=2.884 – after learning by the new scheme. Despite smaller number of tuned parameters, learning by the new scheme proved to be more accurate. Regarding transparency, here using the new scheme resulted in noninterpretability of the extreme term "Heavy" (Figure 10, *a)*. In the new learning scheme, the fuzzy model remained transparent (Figure 10, *b)*.

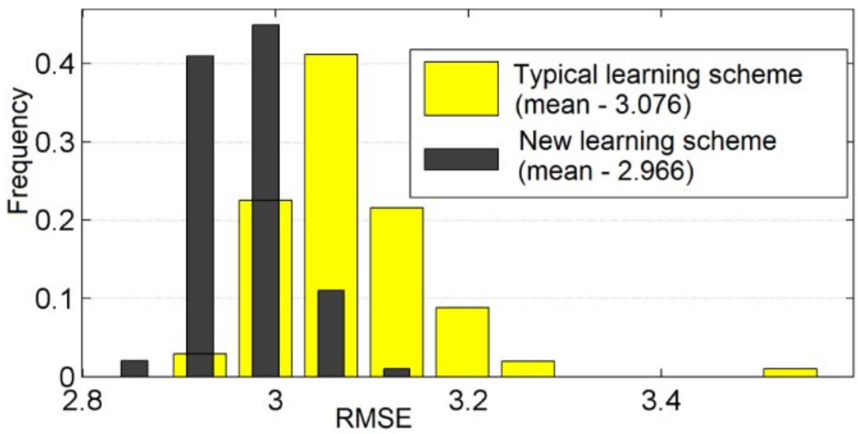

Figure 9

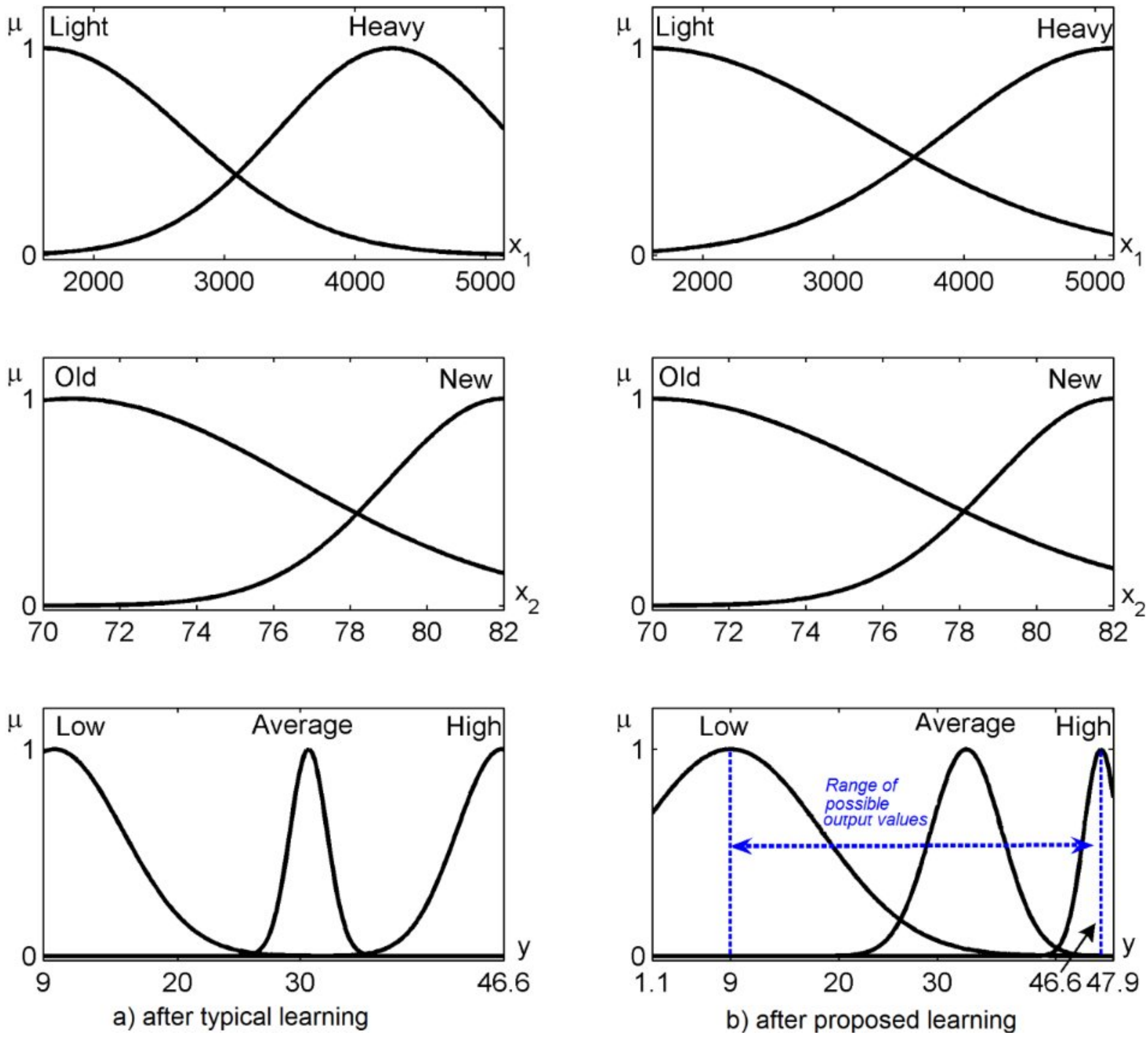

Figure 10

Table 4

No. of rule	After learning by the typical scheme	After learning by the new scheme
1	0.405	1
2	0.994	1
3	0.251	0.159

Conclusions

Requirements of transparency of fuzzy Mamdani model have been formulated. Typical violations of transparency of fuzzy model arising as a side effect of learning by experimental data have been revealed. It is found, that methods of preserving transparency of fuzzy models form a system of hard constraints which does not enable one to attain a high accuracy of learning. We have demonstrated, that ways of enhancing accuracy of learning at the expense of softening constraints somehow break transparency of a fuzzy model. We have suggested a new scheme of learning the Mamdani fuzzy model, which is distinct from the known ones by the following: 1) expansion of the support of fuzzy sets of the output variable; 2) exclusion from the tuned parameters of coordinates of maxima of membership functions of extreme terms; 3) additional constraint for linear ordering of fuzzy terms within the framework of one term-set. Computer simulations indicate, that learning by the new scheme does not break transparency of fuzzy model. Accuracy of fuzzy model is not worse than for a typical learning. Thanks to the possibility of obtaining accurate and transparent models the suggested scheme of learning is useful in identification of nonlinear dependencies with the aid of fuzzy knowledge bases in medicine, economics, biology, sociology and other fields, where both adequacy of the model and meaningful interpretation of its parameters are important.

References

1. Zadeh L., Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. Syst. Man Cybernet.,* 1973, No. 3, 28-44.
2. Mamdani E.H., Assilian S., An experiment in linguistic synthesis with fuzzy logic controller, *Int. J. Man-Machine Studies,* 1975,7, No. 1, 1-13.
3. Nauck D., Kruse R., Neuro-fuzzy systems for function approximation, *Fuzzy Sets and Systems,* 1999,101, No. 2, 261-271.
4. Yager R., Filev D., Essentials of fuzzy modeling and control, John Wiley & Sons, USA, 1994.
5. Shtovba S.D., Design of fuzzy systems by means of MATLAB [in Russian], Goryachaya liniya - Telekom, Moscow, 2007.
6. Rotshtein A.P., Shtovba S.D., Influences of defuzzification methods on the rate of tuning a fuzzy model, Cybernetics and Systems Analysis, 2002, Vol. 38, No. 5, 783–789.
7. Rotshtein A.P., Intelligent technologies of identification: fuzzy logic, genetic algorithms, neural networks [in Russian], UNIWERSUM-Vinnitsa, Vinnitsa, 1999.
8. Rotshtein A.P., Katelnikov D.I., Identification of fuzzy dependencies by fuzzy knowledge bases, Cybernetics and Systems Analysis, 1998, No. 5, 53-61.
9. Casillas J., Cordon O., Jose del Jesus M., Herrera R, Genetic tuning of fuzzy rule deep structures preserving interpretability and its interaction with fuzzy rule set reduction, *IEEE Trans, on Fuzzy Systems,* 2005,**13,** No. 1, 13-29.
10. Alcala R., Alcala-Fdez J., Casillas J., Cordon O., Herrera R, Hybrid learning models to get the interpretability-accuracy trade-off in fuzzy modeling, *Soft Comput.,* 2006, No. 10, 717-734.
11. Babuska R., Construction of fuzzy systems — interplay between precision and transparency, *Proc. of Europ. Sympos. on Intell. Techn.,* Aachen (Germany), 2000, 445-452.
12. Roubos H., Setnes M., Abonyi J., Learning fuzzy classification rules from data, Developments in Soft Computing, Ed. by John R., Birkenhead R., Springer-Verlag, Berlin, 2001, 108-115.
13. Paiva R.P., Dourado A., Merging and constrained learning for interpretability in neuro-fuzzy systems, *Proc. of Europ. Sympos. on Intell. Techn.,* Hybrid Systems and Their Implement, on Smart Adaptive Systems "EUNITE", Tenerife (Spain), 2001, 17-21.
14. Rotshtein A., Design and tuning of fuzzy rule-based system for medical diagnosis, Fuzzy and neuro-fuzzy systems in medicine Ed. by Teodorescu N.H., Kandel A., Jain L.C., CRC-Press, Boca-Raton, 1998,243-289.
15. Miller G.A., The magic number seven plus or minus two: some limits on our capacity for processing information, *Psychological Review,* 1956, No. 63, 81-97.
16. Jimhez R, Gomez-Skanneta A., Roubos H., Babuska R., A multi-objective evolutionary algorithm for fuzzy modeling, *Proc. of Intern. Fuzzy Systems Association and the North American Fuzzy Inform. Proces. Society Joint Conf.* (IFSA/NAFJJPS) Vancouver, Canada, 2001, 1222-1228.
17. Riid A., Transparent fuzzy systems: modeling and control, PhD Thesis. Tallinn Technical University: Department of Computer Control, Tallinn, Estonia, 2002, http://www.dcc.ttu.ee/andri/teosed/tfs-mac.pdf.
18. MPG data base of UCI Machine Learning Repository, http://www.ics.uci.edu/~mlearn/MLRepository.html.