Optimization of a Water Purification Technological Process by Genetic Algorithms

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Abstract. The task of optimizing a water purification technological process involves finding such a process structure choice which provides a necessary level of water quality within prescribed cost limits. In this article this optimization is carried out using genetic algorithms; the resulting solution time is considerable reduced on account of a simultaneous optimum search from various initial points. This is provided by the availability of the initial set of technological process variants upon which the operations of crossover, mutation and selection are based. The efficiency of using genetic algorithms is increasing with the growth of problem complexity and technological process dimension.

1 INTRODUCTION

The quality of water which we consume is directly connected with the cost of treatment. Modern water purification devices (or filters) allow us to simultaneously reduce contamination of different types. Using different water purification devices provides us with the tools to determine the extent to which we may decontaminate the water. In this way we may formulate a problem of optimization, that is to say, to choose a set of water purification process, as which will provide the necessary level of quality at minimum cost. An initial perspective would view this problem as one that may be solved by using known mathematical programming methods [1]. However, taking into account that contamination of many diverse types increases the dimensionality of the state space, it becomes clear that using classical mathematical programming techniques becomes impractical. Therefore, in this article, the task of water purification process optimization is solved using genetic algorithms (GA) [2, 3], which allow find out next to global optimal solution quickly, and besides do not have much mathematical requirements about the optimization problem. Principal distinction of GA methods from classical ones is in the fact that they do not use the notion of a derivative (gradient) while choosing search direction and they are based on crossover, mutation and selection operations.

Note, that GA are already successful applied for optimization the single operation in a technological process of water purification – chlorination [4]. In this article, GA are used for structure optimization of whole technological process of water purification.

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2 PROBLEM STATEMENT

For a mathematical statement of the optimization task we introduce the following notations:

m - number of contamination types (stones, stand, plankton etc.);

n - number of filter types;

x_i - number of type i filters used in water purification technological process ($i=\overline{1,n}$);

$X=\{x_1,x_2,...,x_n\}$ - controlled variables vector, used to determinate the water purification technological process structure;

N^j_{inp} - concentration of type j contamination on input to water purification technological process ($j=\overline{1,m}$);

N^j_{out} - concentration of type j contamination on output from water purification technological process given by vector X ($j=\overline{1,m}$);

C_i - price of type i filter;

$C(X)$ - price of filtering process given by vector X .

According to technological process reliability design theory [5-7], the problem of water purification process optimization consists in finding a vector X for which

$$C(X)\to min \quad and \quad N^j_{out}(X)\le N^j_{ad}\ ,\ \ j=\overline{1,m}\ , \qquad (1)$$

where N^j_{ad} is the threshold admissible concentration of the j-th type contamination.

Relations connecting filtering price and output concentrations of contamination with technological process parameters have the form

$$C(X)=\sum_{i=1}^{n} C_i\cdot x_i\ ,$$

$$N^j_{out}(X)=N^j_{inp}\cdot\prod_{i=1}^{n}(f^j_i)^{x_i}\ , \quad j=\overline{1,m}\ ,$$

where $f_i^j \in [0,1]$ is the coefficient of filtering for type j contamination by type i filter. Here, f_i^j gives the residual, contamination concentration of type j using one filter of type i for case, when this concentration at the filter input equals 1. If a filter of type i does not reduce concentration of a type j contamination at all, then $f_i^j = 1$. If a filter of type i absolutely eliminates the concentration of type j contamination, then $f_i^j = 0$.

Problem (1) actually represents a constrained integer programming problem, which can be solved by genetic algorithms.

3 THE MAIN NOTIONS OF GENETIC ALGORITHMS

GA represents a stochastic method of optimization based on the mechanisms of natural selection acting in live nature [2]. The notions of chromosome, gene and population constitute the base of GA; and classical optimization theory terms of decision variables vector, decision variable and decision set can be brought into correspondence with them.

The basic operations of GA are crossover, mutation and selection:

Crossover represents an operation on two parents-chromosomes yielding two offspring-chromosomes each of which inherits some genes from parents-chromosomes.

Mutation is a random gene modification. Examples of crossover and mutation are shown in Figure 1.

Selection represents itself as some procedure of population formation from the most adapted chromosomes according to its fitness function values.

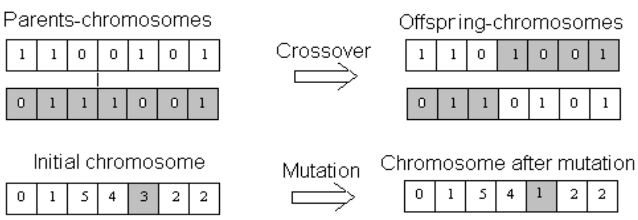

Figure 1. Genetic operations example

Optimization using genetic algorithms consists in performing such a sequence of steps.

1^0. Carrying out genetic coding of decisions variants.

2^0. Generation of an initial population.

3^0. Random way choosing parents and provision of crossover.

4^0. Random way choosing chromosome and provision of mutation.

5^0. Evaluate the values of fitness function for new chromosomes.

6^0. Fixing the best decision.

7^0. Making selection (new population formation) taking into account the fitness function values.

8^0. Repeat steps 3^0-6^0 as many times as necessary.

9^0. Decoding of the decision.

4 FEATURES OF USING GENETIC ALGORITHMS FOR WATER PURIFICATION PROCESS OPTIMIZATION

4.1 Genetic coding of variants

A filtering technological process variant can be represented by chromosome A that contains n genes: $A = \{a_1, a_2, \ldots, a_n\}$. Gene a_i value corresponds to the number of type i filters in the technological process treated. Below we consider example of genetic coding in which the number of filters that can be used in the water purification technological process is $n = 10$. Figure 2 then depicts filtering a technological process variant coding in which two filters of type 1, one filter of type 2 and three filters of type 10 are used.

2	1	0	0	0	0	0	0	0	3
1	2	3	4	5	6	7	8	9	10

Figure 2. Genetic coding example of a water purification process variant

4.2 Crossover and mutation

The uniform crossover and mutation (Figure 1) can be used in GA optimization of a water purification technological process.

4.3 Selection

We propose to carry a selection procedure based on the following mixed strategy:

1) find out the best chromosome and include it into new population;

2) append the remaining chromosomes into new population by spinning the roulette wheel [2, 3].

The first part of the strategy accords to a deterministic selection approach, and the second one is a stochastic approach [2]. The major advantage of described selection strategy, i.e. roulette-wheel with elitism, in compare with uniform roulette wheel is saving the best chromosome on each epoch.

A selection is yielded on enlarged sampling space [2], when both parent and offspring chromosomes have the same chance of competition for survival.

4.4 Fitness function

We propose the fitness function as

$$FF(X) = \frac{1}{C(X)}.$$

In this case the fitness function has the following properties that are need for roulette wheel selection:

- it is a positive function;
- the fitness function value of the best (the worst) chromosome is maximal (minimal).

4.5 Taking into account constrains

Optimization of the water purification technological process is the task of constrained optimization; therefore, the solution must satisfy all constraints (1). Taking into account the constrains we use a penalty function [1]; if a chromosome does not satisfy even one of the constraints, then need to add some negative penalty value to its fitness function. Using the penalty function is the most common technique to handle infeasible solutions in the GA for constrained optimization problems [2].

We propose the penalty function, value of which adds fitness function of some chromosome A :

$$S(A) = pc \cdot \sum_{\forall j:\, N^j_{out}(A) > N^j_{ad}} \frac{N^j_{ad} - N^j_{out}(A)}{N^j_{ad}},$$

where pc is penalty coefficient ($pc > 0$).

5 EXAMPLE

Let us consider a water purification technological process. A database for 26 filters is given in Table 1 [8]. Data about contamination concentration and threshold admissible concentrations are given in Table 2.

Table 1. Filter database

Filter type	Filter name	Purification degree	Price
1	Grating	$f_1^7 = 0.2$	140
2	Sand trap	$f_2^5 = 0.75;\ f_2^7 = 0.85;$ $f_2^9 = 0.05$	350
3	Primary sedimentation tank	$f_3^5 = 0.3;\ f_3^7 = 0.8;\ f_3^9 = 0.1$	400
4	Clarifying agent	$f_4^5 = 0.1;\ f_4^7 = 0.8;$ $f_4^9 = 0.04$	580
5	Pressure filter	$f_5^5 = 0.05;\ f_5^7 = 0.75;$ $f_5^9 = 0.08$	980
6	Hydro cycle	$f_6^4 = 0.6;\ f_6^5 = 0.2;$ $f_6^7 = 0.8;\ f_6^9 = 0.3$	800
7	Micro filters	$f_7^4 = 0$	940
8	Rotary sieve	$f_8^4 = 0.7;\ f_8^5 = 0.02;$ $f_8^7 = 0.75;\ f_8^9 = 0.4$	850
9	Two stream filters	$f_9^4 = 0.75;\ f_9^5 = 0.05;$ $f_9^7 = 0.75;\ f_9^9 = 0.08$	970
10	Wash filters	$f_{10}^4 = 0.2;\ f_{10}^5 = 0.05;$ $f_{10}^7 = 0.8;\ f_{10}^9 = 0.08$	920
11	Chlorine treatment	$f_{11}^1 = 0.05;\ f_{11}^2 = 0;\ f_{11}^{11} = 0$	1100
12	Ozone treatment	$f_{12}^1 = 0;\ f_{12}^2 = 0;\ f_{12}^{11} = 0$	1500
13	Bactericide irradiation of water	$f_{13}^1 = 0.04;\ f_{13}^2 = 0;\ f_{13}^{11} = 0$	1250
14	Reciprocal neutralisation	$f_{14}^{15} = 0.02$	1800
15	Reagents neutralisation	$f_{15}^{15} = 0.02$	2050
16	Active chlorine oxidation	$f_{16}^3 = 0.005;\ f_{16}^{10} = 0.001;$ $f_{16}^{12} = 0.01;\ f_{16}^{14} = 0.001$	2150
17	Air oxygen oxidation	$f_{17}^3 = 0.008;\ f_{17}^{10} = 0.011;$ $f_{17}^{12} = 0.006;\ f_{17}^{14} = 0.01$	2450
18	Electromechanical oxidation	$f_{18}^3 = 0.01;\ f_{18}^{10} = 0.008;$ $f_{18}^{12} = 0.01;\ f_{18}^{14} = 0.007$	2020
19	Radiation oxidation	$f_{19}^3 = 0.22;\ f_{19}^{10} = 0.4;$ $f_{19}^{12} = 0.25;\ f_{19}^{14} = 0.2$	1750
20	Fluorine removal by filtering	$f_{20}^{13} = 0.05$	1500
21	Fluorine removal by sedimentation	$f_{21}^{13} = 0.02$	1900
22	Fluorine removal by hyper filtering	$f_{22}^{13} = 0.03$	1670
23	Lime-soda softening	$f_{23}^6 = 0.75;\ f_{23}^8 = 0.8$	1120
24	Ion exchange softening	$f_{24}^6 = 0.75;\ f_{24}^8 = 0.75$	1000
25	Demineralisation softening	$f_{25}^6 = 0.8;\ f_{25}^8 = 0.68$	1050
26	Coagulation	$f_{26}^4 = 0.75;\ f_{26}^5 = 0.2;$ $f_{26}^9 = 0.03$	1100

Note that only those filtering coefficients that differ from 1 are shown in Table 1. For example, value $f_1^7 = 0.2$ in the first line of this table has the meaning that a filter such as «grating» lowers type 7 contamination («stones») concentration 1/0.2=5 times but does not filter other contamination types.

Genetic optimization was carried out with the following parameters:

- maximum number of filters of same type is 3;
- population size is 20 chromosomes;
- number of crossovers per one epoch is 8;
- number of mutations per one epoch is 14, i.e. mutation ratio is $14 \bullet 100\%/(20 \bullet 26)=2.69\%$;
- total number of epochs (selections) is 1000;
- penalty coefficient is 12.

We found the following optimal chromosome using proposed GA:

2	1	0	1	0	0	1	0	0	0	1	0	0	1	0	0	0	1	1	1	0	0	0	1	0	0

The following filtering scheme corresponds to the above chromosome: *grating (twice) – sand trip – clarifying agent – micro filters – chlorine treatment – reciprocal neutralization – electromechanical oxidation – radiation oxidation – fluorine removal by hyper filtering – ion exchange softening*. The values of contamination concentration after filtering according to the given scheme are presented in the last column of Table 2. The resulting price of this filtering policy is 11320 standard units of currency.

Optimal decision was obtained on the 827-th epoch. Dynamics of optimization by GA was show in Table 3.

Table 2. Contamination concentrations

Conta-mination type	Contamina-tion name	Concent-ration at the input, $\%10^{-3}$.	Threshold admissible concentration, $\%10^{-3}$	Contamination concentration at the output, $\%10^{-3}$.
1	Bacteria	4	0.24	0.2
2	Viruses	5	0.03	0
3	Hydro sulphide	50	0.12	0.11
4	Plankton	40	4	0
5	Silt	360	60	27
6	Calcium	20	18	15
7	Stones	125	10	3.4
8	Magnesium	30	24	22.5
9	Sand	620	5	1.24
10	Hydrogen sulphide	25	0.075	0.008
11	Spore	5	0.14	0
12	Sulphides	8	0.1	0.02
13	Fluor	50	3.2	2.5
14	Cyanides	12	0.032	0.017
15	Alkali	34	0.8	0.68

Table 3. Dynamics of optimization by genetic algorithm

Number of epoch	Best chromosome	Price of filtering
1	1 0 1 0 2 0 2 0 1 1 2 0 2 3 1 3 0 0 1 3 0 1 3 1 1 1	39300
5	1 0 1 0 2 0 1 0 1 1 2 0 2 3 1 2 0 0 1 3 0 1 3 1 1 1	36210
10	1 0 1 0 2 0 2 0 1 1 2 0 2 3 1 2 0 0 1 0 0 1 1 1 1 1	30410
50	1 0 0 0 3 0 1 0 0 1 0 0 1 1 0 1 0 0 1 0 0 1 0 0 1 1	15710
100	1 2 0 0 2 1 1 0 0 0 0 0 1 1 0 1 0 0 1 0 0 1 0 0 1 0	14210
500	2 0 1 0 1 0 1 0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 0 1 0	12120
1000	2 1 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 0 1 0 0	11320

6 CONCLUSION

The main difficulty in using classical methods of mathematical programming for technological process optimization is the high computational requirements connected to the multiextremality of the objective function and to the large number of controlled variables. Application of GA allows cutting the problem solution time on account of carrying out simultaneous optimum searches from various initial points. This is provided by the availability of the initial set of technological process variants on which the operations of crossover, mutation and selection are done. It is difficult to make some theoretic assessment of the gain in labour consumption of technological process optimization problem solution using GA for the time being. Still our experiments show that solution of this problem using traditional search methods will require considerably more computer time. The practicality of using GA is growing with the growth of problem complexity and technological process dimension.

Described GA-based approach of water purification technological process optimization can be used for designing of optimal scheme of filtering another liquids and gases, of waste processing, as well as for optimization of multidimensional technological processes in diverse fields. Typical examples of such processes and semantic interpretations of contaminations and of filters for ones are shown in Table 4.

Table 4. Possible applications of proposed approach

Process type	Contaminations	Filters
Software testing	Bugs of different types	Human- and computer based testing procedures
Product production	Defects of different types	Check and retrofit procedures
Information exchange	Single, double and multiple errors	Algorithms of error recognition and correction

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