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SOFTWARE–HARDWARE SYSTEMS

INFLUENCE OF DEFUZZIFICATION METHODS ON THE RATE OF TUNING A FUZZY MODEL

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The results of computer experiments performed to determine the influence of defuzzification methods on the rate of tuning fuzzy models are presented. The experiments were conducted for the defuzzification methods of the center of gravity and center of maxima and for the median method. The defuzzification method of the center of gravity was found to be the best method providing the highest tuning rate and exactness.

Keywords: *defuzzification, fuzzy set, multiextremal membership function.*

The operation of defuzzification of a fuzzy set, i.e., the transformation of the set into a precise number [1], is a necessary element of identification of nonlinear dependences by means of tuning (training) fuzzy knowledge bases [2–4]. The simplest method of executing this operation is the choice of the precise number corresponding to the maximum of the corresponding membership function [5]. However, the suitability of this method is restricted only to membership functions with one extremum. In the literature on the theory of fuzzy sets, it is proposed to execute the operation of defuzzification of multiextremal membership functions by the methods of the center of gravity and center of maxima and by the median method [1, 6, 7].

Since the criterion of quality of tuning a fuzzy model depends on the defuzzification operation [3], the choice of a method that executes this operation and provides the best rate and accuracy indices of the tuning procedure being used is of interest. This article is an outgrowth of [3] and presents the results of computer experiments in which the investigation of defuzzification methods was interrelated with the quality indices of tuning fuzzy models.

1. A PROBLEM OF TUNING A FUZZY MODEL

Let us consider an object of the form

$$y = f(x_1, x_2, \ldots, x_n) \tag{1}$$

that has n inputs (x_i, $i = \overline{1, n}$) and one output (y) and for which quantitative intervals of changing its inputs $[\underline{x}_i, \bar{x}_i]$ ($i = \overline{1, n}$) and output $[\underline{y}, \bar{y}]$ are known. We assume that the interrelation "inputs-output" can be represented as the following fuzzy knowledge base:

$$
\begin{array}{ll}
& \text{IF } [(x_1 = a_1^{j1}) \text{ AND } (x_2 = a_2^{j1}) \text{ AND} \ldots \text{AND } (x_n = a_n^{j1})] \text{ (with a weight } \alpha_{j1}) \\
& \text{OR } \quad [(x_1 = a_1^{j2}) \text{ AND } (x_2 = a_2^{j2}) \text{ AND} \ldots \text{AND } (x_n = a_n^{j2})] \text{ (with a weight } \alpha_{j2}) \\
\ldots & \text{OR } \quad [(x_1 = a_1^{jk_j}) \text{ AND } (x_2 = a_2^{jk_j}) \text{ AND} \ldots \text{AND } (x_n = a_n^{jk_j})] \text{ (with a weight } \alpha_{jk_j}) \\
& \text{THEN } \quad y = d_j \text{ for all } j = \overline{1, m},
\end{array} \tag{2}
$$

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where a_i^{jp} is a linguistic term used for estimation of a variable x_i in the line whose number is jp $(p=\overline{1, k_j})$, k_j is the number of conjunction lines in which the output y is estimated by a fuzzy term d_j $(j=\overline{1, m})$, α_{jp} is a number that belong to the interval $[0, 1]$ and characterizes the subjective measure of confidence of an expert in the statement whose number is $jp\,(j=\overline{1, m},\ p=\overline{1, k_j})$.

Here, system (2) is considered as a generalization of the knowledge base proposed in [3] for the case where the output variable is estimated by fuzzy terms.

Let $\mu^{jp}(x_i)$ be the function of membership of an input x_i in a fuzzy term a_i^{jp}, $i=\overline{1, n}$, $j=\overline{1, m}$, $p=\overline{1, k_j}$, i.e., we have $a_i^{jp}=\int_{\underline{x}_i}^{\overline{x}_i} \mu^{jp}(x_i)/x_i$, $\mu^{d_j}(y)$ is the function of membership of the output y in a fuzzy term d_j, $j=\overline{1, m}$, i.e., we have $d_j=\int_{\underline{y}}^{\overline{y}} \mu^{d_j}(y)/y$. Then, according to [3], the degree of membership of a concrete input vector $X^*=\{x_1^*, x_2^*, \ldots, x_n^*\}$ in fuzzy terms d_j is specified by the following system of fuzzy logic equations:

$$\mu^{d_j}(X^*) = \max_{p=\overline{1, k_j}} \left\{ \alpha_{jp} \cdot \min_{i=\overline{1, n}} [\mu^{jp}(x_i^*)] \right\}, \quad j=\overline{1, m}. \tag{3}$$

Following [5], we define the fuzzy set corresponding to the input vector X^* as follows:

$$\tilde{y} = \bigcup_{j=\overline{1, m}} \int_{\underline{y}}^{\overline{y}} \min(\mu^{d_j}(X^*),\ \mu^{d_j}(y))/y, \tag{4}$$

where $\bigcup$ is the operation of union of fuzzy sets.

We define the precise value of the output corresponding to the input vector X^* as follows:

$$y = \text{defuz}\,(\tilde{y}), \tag{5}$$

where defuz is the operation of defuzzification of a fuzzy set.

To formalize the terms that belong to a fuzzy knowledge base of the form (2), we will use the functions of membership (which are introduced in [3])

$$\mu^T(x) = \frac{1}{1+\left(\dfrac{x-b}{c}\right)^2} \tag{6}$$

of a variable x in an arbitrary fuzzy term T, where b and c are the following tuning parameters: b is the coordinate of the maximum of the corresponding function, $\mu^T(b)=1$, and c is the coefficient of concentration-expansion of the function. Relations (3)-(6) specify a fuzzy model of object (1); the structure of the model corresponds to knowledge base (2). We write this model in the form

$$y = F(X, A, B, C), \tag{7}$$

where $X=(x_1, x_2, \ldots, x_n)$ is an input vector, $A=(\alpha_1, \alpha_2, \ldots, \alpha_N)$ is the vector of weights of rules (lines) in fuzzy knowledge base (2), $B=(b_1, b_2, \ldots, b_q)$ and $C=(c_1, c_2, \ldots, c_q)$ are the vectors of parameters of tuning membership functions of the form (6) that are assigned to fuzzy terms and estimate the inputs and output of object (1), N is the total number of rules (lines) in (2), q is the total number of terms in (2), and F is the operator that specifies the relations "inputs-output" and corresponds to the use of relations (3)–(5).

Let a training sample be given in the form of the following M pairs of experimental data:

$$\{X_h, y_h\},\ h=\overline{1, M}, \tag{8}$$

where $X_h=(x_1^h, x_2^h, \ldots, x_n^h)$ is the input vector of the hth pair and y_h is the corresponding output.

To find the vector of unknown parameters (A, B, C) that minimize the divergence between the model (7) and experimental (8) outputs of the object, we will use the least-squares method. Then the problem of tuning a fuzzy model is formulated as follows: find a vector (A, B, C) that satisfies the restrictions $\alpha_r \in [\underline{\alpha}_r, \overline{\alpha}_r]$, $r = \overline{1, N}$, $b_\nu \in [\underline{b}_\nu, \overline{b}_\nu]$, and $c_\nu \in [\underline{c}_\nu, \overline{c}_\nu]$, $\nu = \overline{1, q}$, and is such that we have

$$R = \frac{1}{M} \sum_{h=1}^{M} [F(X_h, A, B, C) - y_h]^2 \to \min. \tag{9}$$

It is pertinent to note that a basic difference between the proposed fuzzy model (3)–(5) and a similar model used in [3] is an additional possibility of tuning membership functions of the output variable y in the former model. In this case, the fuzzy model from [3] is obtained from formulas (3)–(5) by using the defuzzification by the method of the center of gravity and the membership functions of the output variable in the form

$$\mu^{d_j}(y) = \begin{cases} 1 \text{ for } y \in [\underline{y} + (j-1) \cdot \Delta, \underline{y} + j\Delta] , \\ 0 \text{ otherwise}, \end{cases}$$

where $\Delta = \dfrac{\overline{y} - \underline{y}}{m}$.

2. METHODS OF DEFUZZIFICATION

The most widespread methods of execution of the defuzzification operation (5) are transformations of a membership function by the methods of the center of gravity and center of maxima and by the median method [1, 6, 7].

The defuzzification of a fuzzy set $\tilde{y} = \int_{[\underline{y}, \overline{y}]} \mu_{\tilde{y}}(y) / y$ by the method of the center of gravity is realized by the formula

$$y = \frac{\int_{\underline{y}}^{\overline{y}} y \cdot \mu_{\tilde{y}}(y) dy}{\int_{\underline{y}}^{\overline{y}} \mu_{\tilde{y}}(y) dy}. \tag{10}$$

A physical analog of formula (10) is the determination of the center of gravity of a flat figure bounded by the coordinate axes and the plot of the membership function of the corresponding fuzzy set.

The defuzzification of the fuzzy set $\tilde{y} = \int_{[\underline{y}, \overline{y}]} \mu_{\tilde{y}}(y) / y$ by the median method consists of finding a number y such that we have

$$\int_{\underline{y}}^{y} \mu_{\tilde{y}}(y) dy = \int_{y}^{\overline{y}} \mu_{\tilde{y}}(y) dy. \tag{11}$$

A geometric interpretation of the median method is the determination of a point that belongs to the abscissa axis and is such that the perpendicular restored at this point divides the area under the curve of the corresponding membership function into two equal parts.

The defuzzification of the fuzzy set $\tilde{y} = \int_{[\underline{y}, \overline{y}]} \mu_{\tilde{y}}(y) / y$ by the method of the center of maxima is realized by the formula

$$y = \frac{\int_G y dy}{\int_G dy}, \tag{12}$$

where G is the set of all the elements from the interval $[\underline{y}, \overline{y}]$ that have the maximal degree of membership in the fuzzy set $\tilde{y}$.

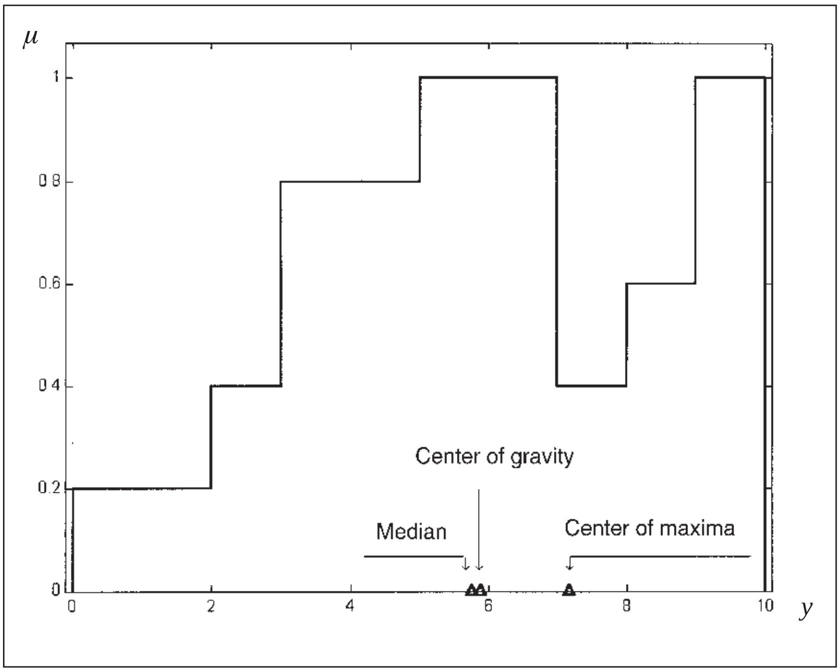

Fig. 1. Defuzzification by different methods.

By the method of the center of maxima, the arithmetic average of the elements of a universal set that have maximal degrees of membership is found. If the set of such elements is finite, then formula (12) can be simplified as follows:

$$y = \frac{\sum_{y_j \in G} y_j}{|G|},$$

where $|G|$ is the cardinality of the set G.

As is seen from this formula, if a membership function has only one maximum, then the coordinate of this maximum is a precise analog of the corresponding fuzzy set.

As an example, we consider the defuzzification of the following fuzzy set by different methods:

$$\tilde{y} = \int_0^2 0.2/y + \int_2^3 0.4/y + \int_3^5 0.8/y + \int_5^7 1/y + \int_7^8 0.4/y + \int_8^9 0.6/y + \int_9^{10} 1/y. \tag{13}$$

The use of the method of the center of gravity (formula (10)) gives the precise number

$$y = \frac{\int_0^2 0.2\cdot y dy + \int_2^3 0.4\cdot y dy + \int_3^5 0.8\cdot y dy + \int_5^7 1\cdot y dy + \int_7^8 0.4\cdot y dy + \int_8^9 0.6\cdot y dy + \int_9^{10} 1\cdot y dy}{\int_0^2 0.2 dy + \int_2^3 0.4 dy + \int_3^5 0.8 dy + \int_5^7 1 dy + \int_7^8 0.4 dy + \int_8^9 0.6 dy + \int_9^{10} 1 dy} = 5.84.$$

The use of the median method (11) gives the precise number $y = 5.8$ since, for this number, the following equality is true:

$$\int_0^2 0.2 dy + \int_2^3 0.4 dy + \int_3^5 0.8 dy + \int_5^{5.8} 1 dy = \int_{5.8}^7 1 dy + \int_7^8 0.4 dy + \int_8^9 0.6 dy + \int_9^{10} 1 dy.$$

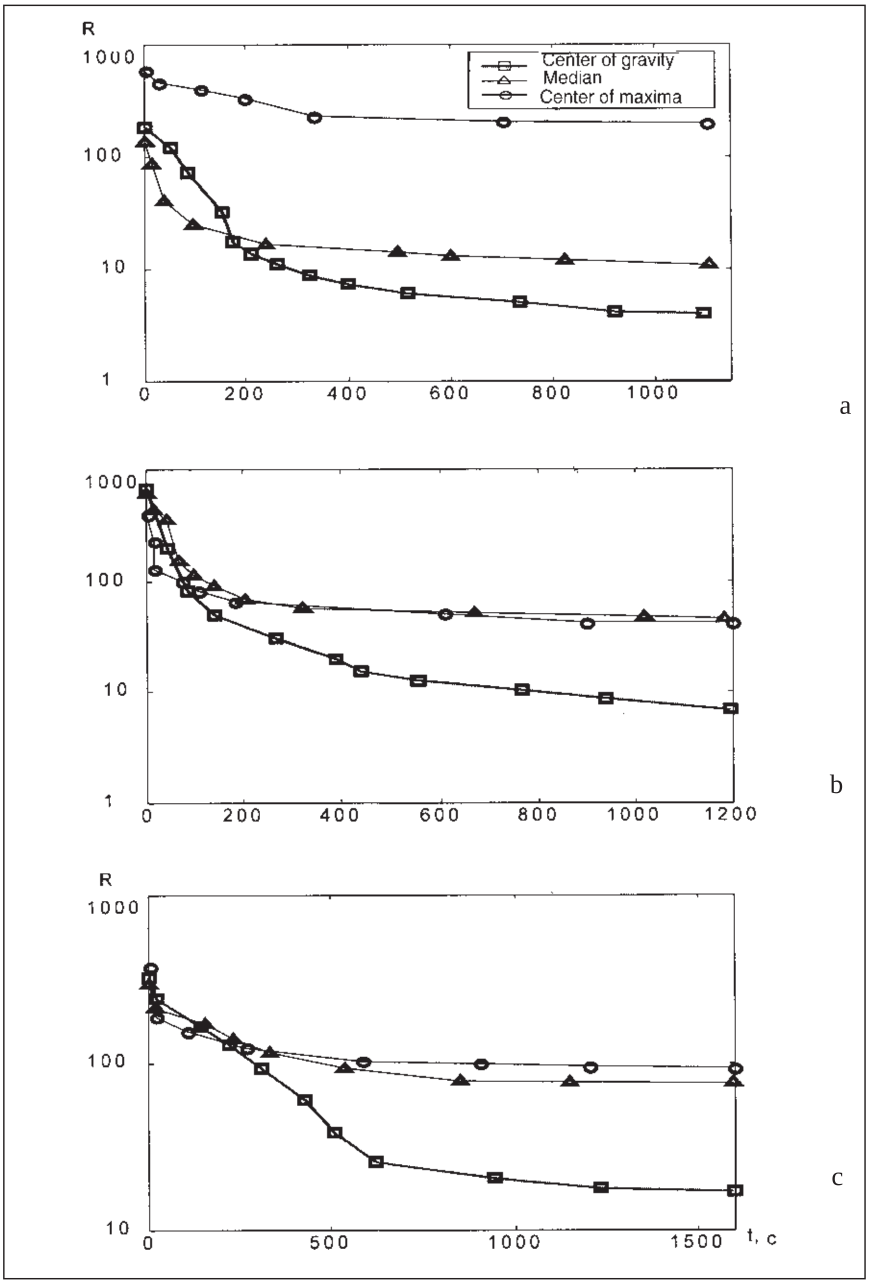

Fig. 2. Training curves of fuzzy models for the following defuzzification methods: (a) a linear reference dependence, (b) a unimodal reference dependence, and (c) a multiextremal reference dependence.

The use of the method of the center of maxima (12) gives the precise number

$$y = \frac{\int_5^7 y dy + \int_9^{10} y dy}{\int_5^7 dy + \int_9^{10} dy} = 7.17.$$

The defuzzification of fuzzy set (13) by various methods is shown in Fig. 1.

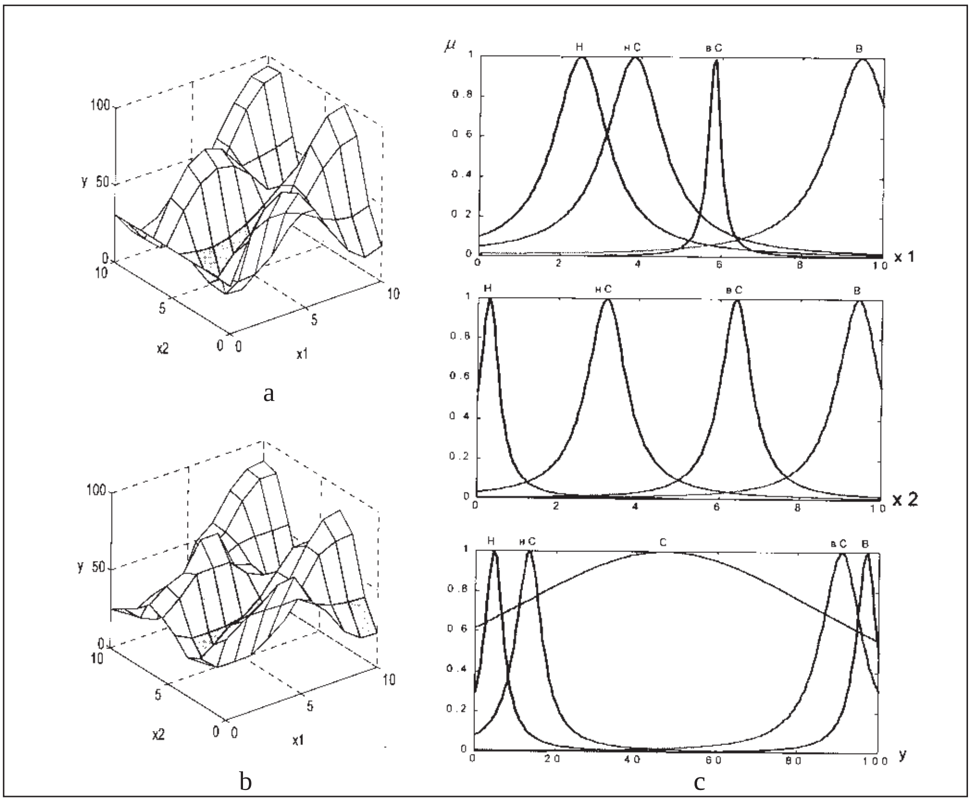

Fig. 3. Identification of dependence (16): (a) a reference dependence, (b) a fuzzy model, and (c) the membership function after tuning.

TABLE 1. A Knowledge Base for Dependence (13)

x_1	x_2	y
L	H	L
L	aA	bA
aA	H	bA
bA	bA	A
aA	aA	A
bA	L	aA
H	bA	aA
H	L	H

TABLE 2. A Knowledge Base for Dependence (14)

x_1	x_2	y
L	bA	L
L	aA	L
bA	aA	L
bA	bA	L
H	bA	bA
H	L	A
L	H	aA
bA	H	aA
aA	H	aA
H	H	H

TABLE 3. A Knowledge Base for Dependence (15)

x_1	x_2	y
L	H	L
bA	H	L
bA	bA	L
H	aA	L
L	bA	L
H	aA	bA
H	L	bA
L	L	A
aA	L	A
aA	bA	A
aA	aA	A
aA	H	A
bA	L	aA
bA	aA	aA
H	bA	H
H	H	H

3. COMPUTER EXPERIMENTS

For generation of fuzzy knowledge bases and training samples, the following three reference models of the form “two inputs–one output” were used:

linear

$$y = 60 + 4x_1 - 6x_2, \quad (14),$$

unimodal

$$y = 0.25((1.7x_1 - 5)^2 + (0.7x_2 - 3)^4), \tag{15}$$

and multiextremal

$$y = 31 + 3x_1 + 40\sin(0.5x_1)\cos(x_2), \tag{16}$$

in which the variables varied within the following intervals: $x_1 \in [0, 10]$, $x_2 \in [0, 10]$, and $y \in [0, 100]$.

In constructing the above fuzzy knowledge bases, the following denotations were used: L means "low," bA means "below the average," A means "average," aA means "above the average," and H means "high."

To dependences (14)–(16) correspond the fuzzy knowledge bases presented in Tables 1–3. These knowledge bases were specified by experts on the basis of the plots of dependences (14)–(16). During the experiments conducted, the fuzzy terms from the knowledge base (Table 1–3) were tuned for each method of defuzzification specified by formulas (10)–(12). At the same time, training curves were constructed in the form of dependences of the accuracy of tuning (*R*) on the training period (*t*). The experiments were carried out using the MatLab package on a personal computer with a processor Pentium-166. For each experiment, the training sample equaled 100 pairs "inputs-output."

As a result of the experiments carried out, we reveal that the defuzzification by the method of the center of gravity (Fig. 2) provides the largest rate and accuracy indices of tuning the considered fuzzy models. As an example, the results of identification of object (16) are presented in Fig. 3.

The defuzzification of a fuzzy set, i.e., its transformation into a precise number, is a necessary element of construction of applied fuzzy systems based on fuzzy logic. In this article, the results of computer experiments with reference dependences are described, in which the rate of tuning (training) fuzzy models was investigated for the above-mentioned defuzzification methods, namely, for the methods of the center of gravity and center of maxima and for the median method. The computer experiments performed by the authors provide reason enough to assume that the best defuzzification method in constructing applied fuzzy systems is the method of the center of gravity.

REFERENCES

1. H.-J. Zimmermann, Fuzzy Set Theory and Its Applications, Kluwer, Dordrecht (1996).
2. A. Rotshtein, "Design and tuning of a fuzzy rule-based system for medical diagnosis," in: N.H. Teodorescu (ed.), Fuzzy and Neuro-Fuzzy Systems in Medicine, CRC-Press (1998), pp. 243-289.
3. A. P. Rotshtein and D. I. Katel'nikov, "Identification of nonlinear objects by fuzzy knowledge bases," Kibern. Sist. Anal., No. 5, 53-61 (1998).
4. A. P. Rotshtein, E. E. Loiko, and D. I. Katel'nikov, "Prediction of the number of diseases on the basis of expert-linguistic information," Kibern. Sist. Anal., No. 2, 178-185 (1999).
5. L. A. Zadeh, The Concept of a Linguistic Variable and Its Application to Approximate Reasoning [Russian translation], Mir, Moscow (1976).
6. K. Asai, D. Vatada, et al., Applied Fuzzy Systems [Russian translation], Mir, Moscow (1993).
7. R. Yager and D. Filev, Essential of Fuzzy Modeling and Control, Wiley, New York (1994).